CONTINUOUS COMPOSITE STEEL CONCRETE BEAMS WITH PARTIAL SHEAR CONNECTION

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Summary: *The paper presents the calculation of continuous composite steel-concrete beams with a partial shear connection and ductile shear connectors. The algorithm for determining the ultimate load as a function of shear resistance capacity is explained. The calculation is applied to a numerical example. The obtained results are discussed. Guidelines for further research in this area are given.*

Keywords: *Partial shear connection, ductile shear connectors, bending resistance, contiuous composite steel-concrete beams*

1. INTRODUCTION

Composite steel-concrete beams are in increasing use over the last several decades. A composite steel and concrete beam's behaviour is characterized by the properties of a shear connection at the concrete slab and the steel beam interface. Depending on the shear connection's strength, shear connections are classified as full and partial [1]. The shear connection is full when the number of connectors allows a full moment of resistance to be achieved in the critical section. On the other side, when fewer shear connectors are used, the shear connection is partial and the corresponding ultimate load is smaller. The use of the partial shear connection is common when the ultimate strength of the composite section does not govern the design. For example, when the composite beam's stiffness is determined from the deflection criteria or, in unpropped construction, when dimensions of the steel beam are determined from a critical stage during construction [1]. According to the shear connectors' ductility and the load-slip characteristics, connectors are classified as ductile and non-ductile [1]. The ductile shear connectors have sufficient deformation capacity to satisfy the assumptions of the plastic design methods. In this paper, the analysis of continuous composite beams with partial shear connection and ideal ductile shear

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connectors is considered. The calculation algorithm is based on the previously developped theoretical model by Stark [2].

2. THEORETICAL MODEL

The use of partial shear connection in continuous steel concrete beams is not explored well in the past, while there are number of its advantages. These were studied by Stark [2] and he developed the theoretical model based on the ideal-plastic material behavior when applying ideal ductile shear connectors. In this paper, this model is slightly modified to adjust to the Eurocode 4 design guide and new "exact" expressions for calculating beam ultimate load are derived. The model is then implemented in Matlab code that allows simple analysis of the continuous composite beams. In order to explain the theoretical model, an example of a continuous beam over three supports and two equal spans is considered [2], Fig. 1. The load consists of a uniformly distributed load, simulated by four concentrated forces per span. The ultimate load depends on the moments of resistance in the sagging and in the hogging moment sections. The relation is given by Eq. 1.

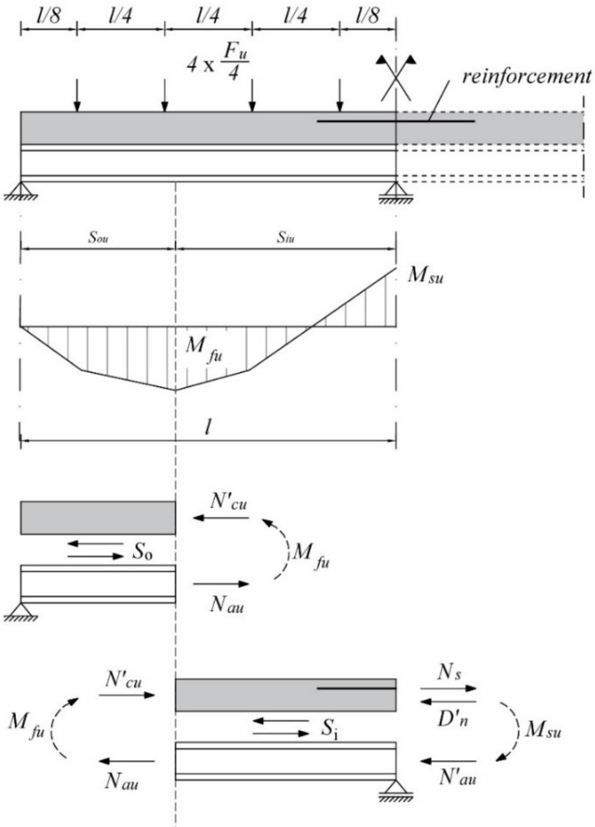

Figure 1. Continuous beam with partial shear connection [2]

$$F_u = \frac{8M_{fu} + 3M_{su}}{l}, \tag{1}$$

M_{fu} - sagging (positive) moment of resistance
M_{su} - hogging (negative) moment of resistance
l - continuous beam span

The longitudinal shear forces, denoted by S_o and S_i, between the concrete slab and the steel beam significantly influence the ultimate load of the continuous beam with partial shear connection [2]. Here, S_o is the longitudinal shear force between the maximum sagging moment and the end support (see Fig. 1), and S_i represents the longitudinal shear force between the maximum sagging moment and the intermediate support. The shear connection resistances are denoted by S_{ou} and S_{iu} for the external and the internal span, respectively [2]. The relation between these two longitudinal shear resistances is not known in advance, which makes the calculation more complicated. In order to achieve that both shear planes are critical (i.e. external and internal shear span), it is necessary to find the optimal values for S_{ou} and S_{iu}. According to the Fig. 1, the necessary equilibrium conditions are derived (Eqs. 2 - 4).

$$N'_{cu} = S_o = S_{ou} \tag{2}$$

$$D'_n = N_s - S_{iu} + S_{ou} \tag{3}$$

$$N'_{cu} + N_s - S_{iu} - D'_n = 0 \tag{4}$$

The external moment of resistance M_{fu} is defined according to the stress distribution, as shown in Fig. 2. Both materials are assumed to behave as ideal plastic materials. So the strains are not limited. The design strength of concrete in the idealized diagram is assumed to have a reduced value kf_{cd} ($k = 0.85$) [3]. The calculation of the moment of resistance is dependent on the position of the neutral axis. Plastic stress distribution with the neutral axis situated in the steel beam is given in Fig. 2.

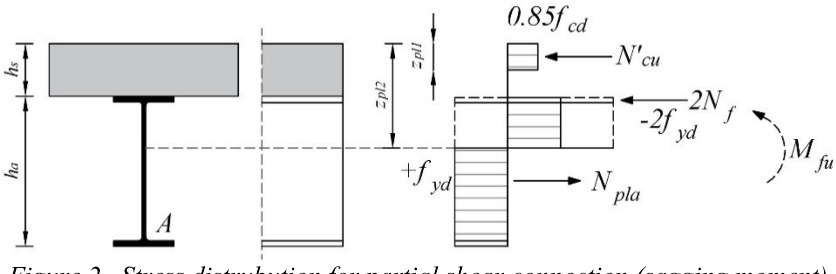

Figure 2. Stress distrubution for partial shear connection (sagging moment)

The plastic moment of resistance is determined by the reduced value of the compressive force in the concrete slab. The position of the plastic neutral axis in the concrete slab - z_{pl1} is determined by force N_{cu}' (Eq. 5). The value of this force depends on the degree of shear connection. At the same time, inside the steel profile, another plastic neutral axis - z_{pl2} determines the stress distribution of the composite cross-section (Eq. 6).

$$z_{pl1} = \frac{N_{cu}'}{0.85 \cdot f_{cd} \cdot b_{eff}} \tag{5}$$

$$z_{pl2} = h + t_{ft} + \frac{N_{pla} - N_{cu}' - 2 \cdot N_{ft}}{2 \cdot f_{yd} \cdot t_w} \tag{6}$$

If the position of the neutral axis is in the web of the steel profile, the moment of resistance is obtained by Eq. 7.

$$\begin{aligned} M_{fu} &= N_{pla} \cdot (y_a + h - \frac{z_{pl1}}{2}) - 2 \cdot N_{ft} \cdot (h + \frac{t_{ft}}{2} - \frac{z_{pl1}}{2}) - \\ &-2 \cdot f_{yd} \cdot t_w \cdot (z_{pl2} - h - t_{ft}) \cdot (z_{pl2} - \frac{z_{pl2} - h - t_{ft}}{2} - \frac{z_{pl1}}{2}) \end{aligned} \tag{7}$$

f_{cd} - design compressive strength of concrete
f_{yd} - design yield strength of steel
b_{eff} - the effective width of the concrete slab
N_{cu}' - reduced value of the compressive force in the concrete slab
N_{pla} - force in the steel section obtained by the steel yield strength
$N_f = N_{ft} = N_{fb}$ - force in the flange of the steel section (symmetrical cross-section)

The different position of the neutral axis is taken into account for obtaining the value of M_{fu}. The following part of the calculation refers to the internal shear span and determining the hogging moment of resistance M_{su} as shown in Fig. 3.

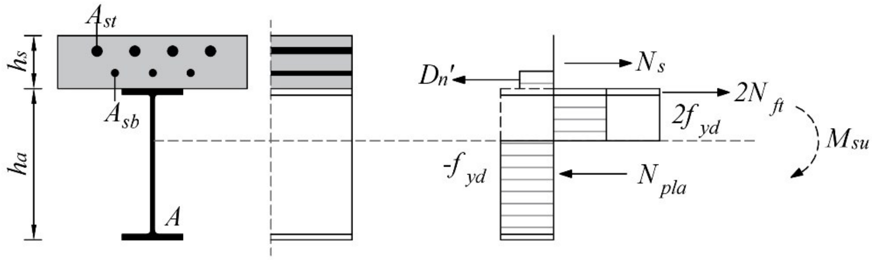

Figure 3. Stress distrubution for partial shear connection (sagging moment)

At failure, the reinforcement always yields so that $N_s = A_s \cdot f_{sy}$ (f_{sy} - reinforcement yield strength). In order to meet the equilibrium conditions based on Fig. 1 in the interior shear span, a force D_n' is introduced inside the concrete part of the section, which is limited at the top and must be positive because the tensile stresses are neglected in concrete.

In case when force D_n' is equal to zero, a full shear connection is achieved between the concrete slab and steel cross-section. The theoretical maximum value of D_n' is reached when the force in the steel beam is equal to a tensile force $A \cdot fyd$. The value D_n' is also limited by the condition that, assuming ideal plastic material behaviour, the height of the compression zone can only be equal to the distance between the bottom surface of the reinforcement and the bottom surface of the concrete slab (Eq. 8).

$$\begin{aligned} D_{n,max1}' &= A_a \cdot f_{yd} + N_s \\ D_{n,max2}' &= b_{eff} \cdot (h-e) \cdot f_{cd} \cdot 0.85 \end{aligned} \tag{8}$$

h - composite cross-sectional height
e - distance between the center of reinforcement and the upper surface of the concrete slab

The position of the neutral axis z_{pl1}, z_{pl2} and the hogging moment of resistance M_{su} are calculated according to Eqs. 9 - 11.

$$z_{pl1} = h - \frac{D_n'}{0.85 \cdot f_{cd} \cdot b_{eff}} \tag{9}$$

$$z_{pl2} = h + t_{ft} + \frac{N_{pla} + D_n' - N_s - 2 \cdot N_{ft}}{2 \cdot f_{yd} \cdot t_w} \tag{10}$$

$$\begin{aligned} M_{su} &= N_{pla} \cdot (y_a + h) + D_n' \cdot (h - 0.5 \cdot \frac{D_n'}{0.85 \cdot f_{cd} \cdot b_{eff}}) - Ns \cdot e \\ &-2 \cdot N_{ft} \cdot (h + \frac{t_{ft}}{2}) - 2 \cdot f_{yd} \cdot t_w \cdot (z_{pl2} - h - t_{ft}) \cdot (z_{pl2} - \frac{z_{pl2} - h - t_{ft}}{2}) \end{aligned} \tag{11}$$

The problem is complex when the values of the longitudinal shear resistance S_{ou}, S_{iu} and force D_n' are unknown. As stated above, besides, the relation between S_{ou} and S_{iu} is not known in advance. Therefore, the algorithm for obtaining the ultimate load of a continuous composite steel concrete beams with partial shear connection is explained below.

To obtain the value of ultimate load capacity of composite beam, F_u, it is initially necessary to assume the value of the shear resistance S_{iu}. In addition to the chosen value of S_{iu}, also a value for D_n' is chosen, so the Eq. 4 directly leads to a value for N_{cu}'. The value of S_{ou} follows from the equilibrium condition for the external shear span (Eq. 2). In the written Matlab program, for precisely defined material and geometric characteristics of beam, for each value of force D_n' in the range [0, $D_{n,max}'$], values for M_{su}, M_{fu}, and F_u are calculated. In that way, a series of F_u values were obtained, of which the maximum value is marked as ultimate load $F_{u,\max}$. Then, the corresponding value of N_{cu}' can be determined. From the condition $N_{cu}' = S_o = S_{ou}$ now follows the minimum required shear resistance in the external shear span. This procedure is illustrated on the following numerical example.

3. NUMERICAL EXAMPLE

The numerical example refers to continuous composite steel-concrete beams with partial shear connection with geometric characteristics in Fig. 4 experimentally tested by Stark [2]. The beams are subjected to four concentrated forces per span. Tests labelled as FA2, FA3, FA4, FB6, FB7 and FB8 are discussed. The test beams differ in the degree of shear connection in the external and internal shear span, as shown, in a percentage, in Table 1. The measured values of material characteristics from Table 1 are taken into account.

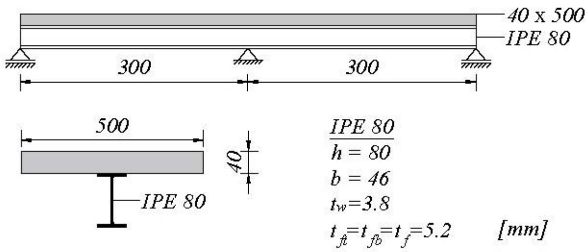

Figure 4. Dimensions of beams and cross-section

The connectors are assumed to be ideally ductile. The amount of reinforcement is taken in the value of $As = 0.5 \cdot A \cdot f_{yd} / f_{sy}$ or it is taken as 0.5% of the cross-section of the concrete slab ($As = 0.005 \cdot 40 \cdot 500 = 100 mm^2$). The position of the reinforcement above the middle support, according to the parameter $e = 11\ mm$ is also defined.

Table 1. Dimensions and materials properties [2]

Beam	S_{ou} [%]	S_{iu} [%]	f_{ck} [MPa]	E_c [GPa]	f_y [MPa]	A [mm^2]	f_{ys} [MPa]	A_s [mm^2]
FA2	99	76	36	28.5	336	765	440	254
FA3	119	47	36	28.5	280	764	440	254
FA4	49	44	36	28.5	339	764	440	254
FB6	112	113	37	29	354	764	440	99
FB7	94	87	37	29	336	764	440	99
FB8	50	48	38	28.5	300	768	440	254

The results of the presented theoretical model and the values obtained by experimental analysis are given in the Table 2. The value of the force S_{iu} is assumed to be in a wide range of values. The optimal value of the force S_{ou} for different value of the force S_{iu} indicates the minimum value of the shear resistance for the external span needed to use at all the shear resistance in the internal shear span. The value of the force S_o is determined

by the number of connectors in the model that was tested experimentally. If the shear force S_o is smaller than the resistance S_{ou}, than the shear plane in the external shear span is not critical.
According to these results, it can be concluded that the theoretical model gives acceptable accuracy for ultimate load values. Obtaining the extreme value of the force F_u as the ultimate load is graphically shown in Fig. 5.

Table 2. Comparison of the theoretical and experimental values

Beam	*Experimental value*					*Theoretical value*			
	S_{iu}	S_o	M_{fu}	M_{su}	F_u	S_{ou}	M_{fu}	M_{su}	$F_{u,max}$
	[kN]	[kN]	[kNm]	[kNm]	[kN]	[kN]	[kNm]	[kNm]	[kN]
FA2	280.5	255	16.8	11.4	55.5	252	17.86	11.44	59.45
FA3	153	255	13.9	7.9	43.52	139.1	13.12	9.58	45.96
FA4	161.5	127.5	15.1	12.1	50.71	166.9	15.64	10.02	52.34
FB6	306	255	16.6	7.7	50.2	281.1	17.41	8.53	55.1
FB7	272	255	18.5	10.4	58.18	265.6	18.76	9.2	59.39
FB8	144.5	127.5	14.8	7.6	46.73	155.5	15.26	9.52	50.97

Therefore, for each value of S_{iu} and N_s exists an unique value of S_{ou} which leads to a maximum ultimate load F_u. When the shear resistance in the external shear span is chosen larger than optimum value of S_{ou}, the ultimate load will not become larger than $F_{u,\max}$.
It should be mentioned that according to Eurocode 4 [3] for composite continuous beams, a full shear connection is required above the middle support. Therefore, it is necessary to assume the value of the force $D_n' = 0$ when determining hogging moment of resistance M_{su}. It is interesting to compare the ultimate load for the previously explained procedure $F_{u,p}$ and the requirements of the Eurocode 4 [3] - $F_{u,f}$, Table 3.

Table 3. Ultimate load for partial and full shear connection in the internal shear span

Beam	FA2	FA3	FA4	FB6	FB7	FB8
$F_{u,p}$[kN]	59.45	45.96	52.34	55.1	59.39	50.97
$F_{u,f}$[kN]	54.66	35.49	42.1	54.43	58.12	38.86

Table 3 shows that the ultimate load values when assuming a full shear connection in the internal shear span of the beam are smaller. The differences are larger in the beams where the optimal value of the force D_n' was higher. These dependencies will be investigated in further work. The presented procedure can also be applied to beams with non-ductile connectors.

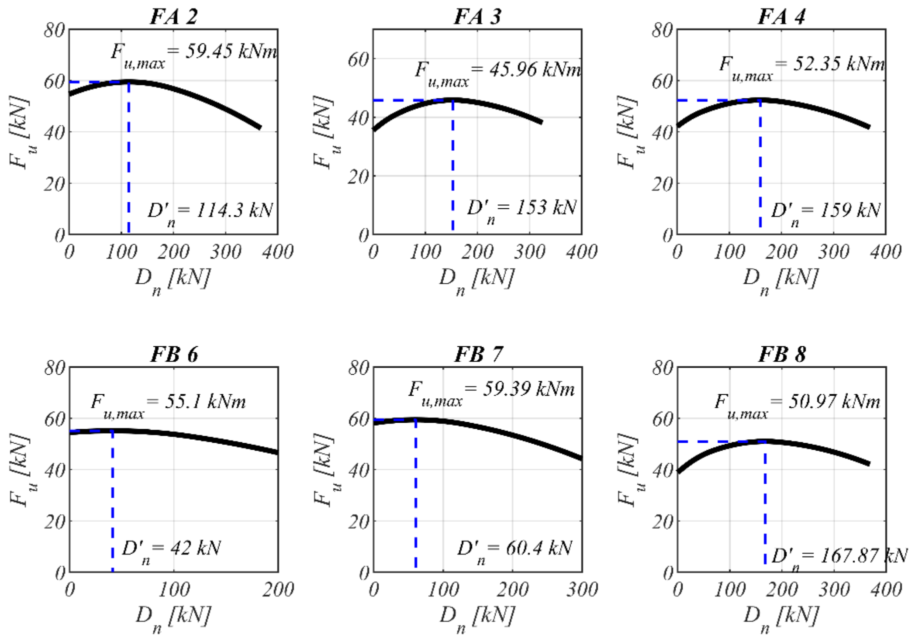

Figure 5. Graphical interpretation of the theoretical model

4. CONCLUSION

The paper discusses the numerical model for calculation of ultimate loading of steel-concrete composite beams with ductile shear connectors. The considered problem requires the optimization and balance between the connection resistances for the external and the internal span. The model has been implemented in Matlab code and used in numerical calculations. Comparing the available experimental and numerically obtained results, it is shown that model predicts with high accuracy the ultimate loading for the continuous composite steel-concrete beams.

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КОНТИНУАЛНЕ СПРЕГНУТЕ ГРЕДЕ ОД ЧЕЛИКА И БЕТОНА СА ПАРЦИЈАЛНИМ СМИЧУЋИМ СПОЈЕМ

Резиме: *У раду се разматра прорачун континуалног спрегнутог гредног носача од челика и бетона са парцијалним смичућим спојем и дуктилним спојним средствима (можданицима). Дат је поступак за одређивање граничног оптерећења у функцији подужне смичуће носивости (отпорности). Објашњен теоријски модел је приказан на бројном примеру. Дискутовано је о резултатима и дати су правци даљег истраживање на ову тему.*

Кључне речи: *парцијални смичући спој, дуктилни можданици, носивост на савијање, континуалне спрегнуте*