STABILITY ANALYSIS OF A LEVELLING NETWORK AS A PRELUDE TO A RESIDENTIAL BUILDING CONSTRUCTION

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Summary: *The significance of geodetic deformation monitoring during the construction of any type of civil engineering structure is a well-known matter. Many classical and sophisticated geodetic monitoring methods employing total station, GNSS, laser scanning, photogrammetric and other technologies have been developed and are actively in use. Nevertheless, precise geometric levelling is still one of the most accurate of all geodetic measurement methods. This paper presents a practical example of deformation analysis being applied to a monitoring levelling network established for the determination of vertical displacements of a residential building in Zagreb in the Republic of Croatia. Before the construction start, the stability of the monitoring network has been analyzed in two measurement epochs and vertical displacements of the network's benchmarks have been determined by applying the Karlsruhe deformation analysis method. All relevant results are presented.*

Keywords: *levelling network, vertical displacements, Karlsruhe method, residential building, construction*

1. INTRODUCTION

Geodetic deformation monitoring of civil engineering structures during and after construction is one of the most challenging geodetic tasks of high importance. The stability of the structure under construction and its surroundings, compliance with construction projects, early warning in case of unexpected behavior and other safety aspects are just some of the key challenges of a geodetic engineer on a construction site. The catastrophic collapses of the Gleno Dam in Italy in 1923 and of the St. Francis Dam

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in the USA in 1928, where key triggers for the gradual development of geodetic deformation monitoring methods [1]. To date, many classical and sophisticated monitoring methods employing total station, GNSS, laser scanning, photogrammetric and other technologies have been developed and are in use. Conventional deformation analysis involves spatial-temporal modelling of the analyzed object, subjected to deformation processes, by discretization with a finite number of characteristic points in such a way that their change in position depict the displacements and deformations of the whole object. Geodetic measurements at specified time intervals (epochs) allow temporal modeling of these deformation processes. The most notable methods are Hannover, Karlsruhe, Delft, Fredericton, and Munich [1,2]. Practical applications of the stated methods can be found in [1-9].

This research presents the measurement and deformation analysis procedure of a precise levelling monitoring network established for the construction of a residential building in Zagreb in the Republic of Croatia. The levelling network is established with its main purpose to serve as a foundation, i.e., height basis for vertical displacement determination of the planned residential building to be constructed and its existing surrounding buildings during all construction phases. Because of the recent seismic activity of the wider Zagreb area [10] the stability of the preestablished network has been analyzed before the construction start. The established precise levelling network has been measured in two epochs and by utilizing the Karlsruhe method of deformation analysis its stability has been tested and, as a result, vertical displacements of the network points (benchmarks) have been quantified.

2. THEORETICAL OVERVIEW OF THE KARLSRUHE METHOD

The Karlsruhe method of deformation analysis was developed by multiple authors at the Geodetic Institute of the University of Karlsruhe (present day Karlsruhe Institute of Technology) in the late 1970's and early 1980's [5,11]. This paper elaborates the method's theoretical background in a concise manner. For detail explanation refer to [1-5,11]. The method is based upon independent adjustments of zero (reference) and control (*i*-th) epochs and on their joint adjustment. In short, the algorithm of the Karlsruhe deformation analysis method applied on a geodetic monitoring network can be divided into three basic steps [1,3,5,11]:

1. the individual least square adjustments of the geodetic network in the reference and control epochs using the minimum trace datum with outlier detection and localization,
2. the joint least square adjustment of the geodetic network in both analyzed epochs and determination of unstable points by means of general linear hypothesis testing,
3. final network least square adjustment and graphical interpretation of the gained results.

Within the first deformation analysis step, i.e., for every individual network adjustment, a square form Ω_i is determined and the joint square form Ω_0, which contains the information on random measurement errors, is calculated as [1,3-5,11]:

$$\Omega_0 = \sum_{i=0}^{k} \Omega_i = \sum_{i=0}^{k} \mathbf{v}_i^{\mathrm{T}} \mathbf{P}_i \mathbf{v}_i \, , \tag{1}$$

where:

$\mathbf{v}_i$ – vector of residuals,

$\mathbf{P}_i$ – weight matrix of observations,

k – number of measurement epochs.

The total number of freedom degrees b is determined as the sum of the degrees of freedom b_i from network adjustments of individual epochs [1,3-5,11]:

$$b = \sum_{i=0}^{k} b_i \,. \tag{2}$$

In the second deformation analysis step the measurement values in both epochs (reference and control) are jointly adjusted where the vector of unknown parameters $\mathbf{x}$ is divided into three sub-vectors as follows [1,3-5,11]:

$$\mathbf{x}^{\mathrm{T}} = \begin{bmatrix} \mathbf{x}_{\mathrm{S}} & \mathbf{x}_{\mathrm{U}}^{'} & \mathbf{x}_{\mathrm{U}}^{''} \end{bmatrix}^{\mathrm{T}}, \tag{3}$$

where:

$\mathbf{x}_{\mathrm{S}}$ – sub-vector of conditionally stable network points,

$\mathbf{x}_{\mathrm{U}}^{'}$ – sub-vector of assumably unstable network points in the reference epoch,

$\mathbf{x}_{\mathrm{U}}^{''}$ – sub-vector of assumably unstable network points in the control epoch.

The design matrix **A**, vector of observations **l**, and the weight matrix of observations $\mathbf{P}$ of the mathematical model of adjustment are all accordingly structurally reorganized. From this joint adjustment, the square form Ω_{J} is determined, containing the information on random measurement errors and movements of unstable network points [1]. Therefore, to determine the square form Ω_{h}, that will contain only the information on point movements, i.e., unstable network points, Ω_{J} and Ω_0 are subtracted [1,3-5,11]:

$$\Omega_{\mathrm{h}} = \Omega_{\mathrm{J}} - \Omega_0 \,. \tag{4}$$

Furthermore, to statistically test the stability of the conditionally stable points contained in the sub-vector $\mathbf{x}_{\mathrm{S}}$, a test statistic F is formed [1,3-5,11]:

$$F = \frac{\Omega_{\mathrm{h}} / f}{\Omega_0 / b} \,; F_{f,b,1-\alpha} \,, \tag{5}$$

$$f = (k-1) \cdot n \cdot p_0 - d \,, \tag{6}$$

where:

$F_{f,b,1-\alpha}$ – F distribution fraction with f and b freedom degrees and significance level α,

k – number of epochs,

n – geodetic network dimension,

p_0 – number of conditionally stable points,

d – rank defect of the design matrix $\mathbf{A}$.

If $F \le F_{f,b,1-\alpha}$, all the conditionally stable points located in the sub-vector $\mathbf{x}_{\mathrm{S}}$ are indeed stable. Otherwise, if $F > F_{f,b,1-\alpha}$, unstable points are present, and it is necessary to localize them. Thus, a set of p_0 partial joint adjustments of the monitoring network follow wherein each adjustment one conditionally stable point is successively excluded together with all observations connected to that network point. As part of each of the p_0 partial joint adjustments, the square form Ω_{J} is determined (in total p_0 square forms). The adjustment providing the minimal value of the stated square form implies that the point excluded in that adjustment must be considered unstable and is moved from the sub-vector $\mathbf{x}_{\mathrm{S}}$ into sub-vectors $\mathbf{x}_{\mathrm{U}}^{'}$ and $\mathbf{x}_{\mathrm{U}}^{''}$. Structural reorganization of $\mathbf{A}$, $\mathbf{l}$ and $\mathbf{P}$ is therefore needed. According to equation (5), the general linear hypothesis is tested and a conclusion on the stability of points remaining in $\mathbf{x}_{\mathrm{S}}$ is made. The explained iterative process of network adjustments and hypothesis testing is conducted until the condition $F \le F_{f,b,1-\alpha}$ is fulfilled. If another iteration of the second deformation analysis step is needed, a new set of $(p_0 - 1)$ partial joint adjustments is conducted. After all unstable network points have been statistically detected, the final joint adjustment is conducted wherein all network points are organized in the vector of unknown parameters $\mathbf{x}$ as stated in equation (3). In the final deformation analysis step the displacements of the unstable monitoring network points are calculated and their significance once again statistically tested. For every unstable network point the test statistics T_j is formed [1,3-5,11]:

$$T_j = \frac{\theta_j^2}{\bar{s}^2} = \frac{\mathbf{d}_j^{\mathrm{T}} \mathbf{Q}_{\mathbf{d}_j}^{-1} \mathbf{d}_j}{m\bar{s}^2}; F_{m,f,1-\alpha}, \tag{7}$$

where:

$\mathbf{d}_j = \mathbf{x}_{\mathrm{U}_j}^{''} - \mathbf{x}_{\mathrm{U}_j}^{'}$ – displacement vector of network point *j*,

$\mathbf{Q}_{\mathbf{d}_j}$ – cofactor matrix of displacements in $\mathbf{d}_j$,

m – network dimension,

$\bar{s}^2 = \dfrac{b_1\bar{s}_1^2 + b_2\bar{s}_2^2}{b_1 + b_2}$ – a posteriori total reference variance calculated as a weighted mean based on the reference variances $\bar{s}_1^2$ and $\bar{s}_2^2$ of individual network adjustments.

If $T_j > F_{m,f,1-\alpha}$, it is statistically conformed that the network point *j* is unstable and has change its position between the two epochs.

3. MEASUREMENT METHODOLOGY AND RESULTS

The construction site, within which the construction of a built-in five-story residential building is planned, is in Medulićeva Street in Zagreb in the Republic of Croatia (Figure

1). There are existing adjacent residential buildings on the north and south side of the planned construction site. The south building is of recent construction date and mostly reinforced concrete structure, but the northern building was built at the beginning of the 20th century and is of masonry structure. Therefore, in terms of safety and the stability of the building under construction and its adjacent buildings (especially the building from the north side) deformation monitoring during all construction phases is extremely important. Thus, a levelling monitoring network surrounding the construction site has been established (Figure 1) with its main purpose is to serve as a height basis for vertical displacement determination.

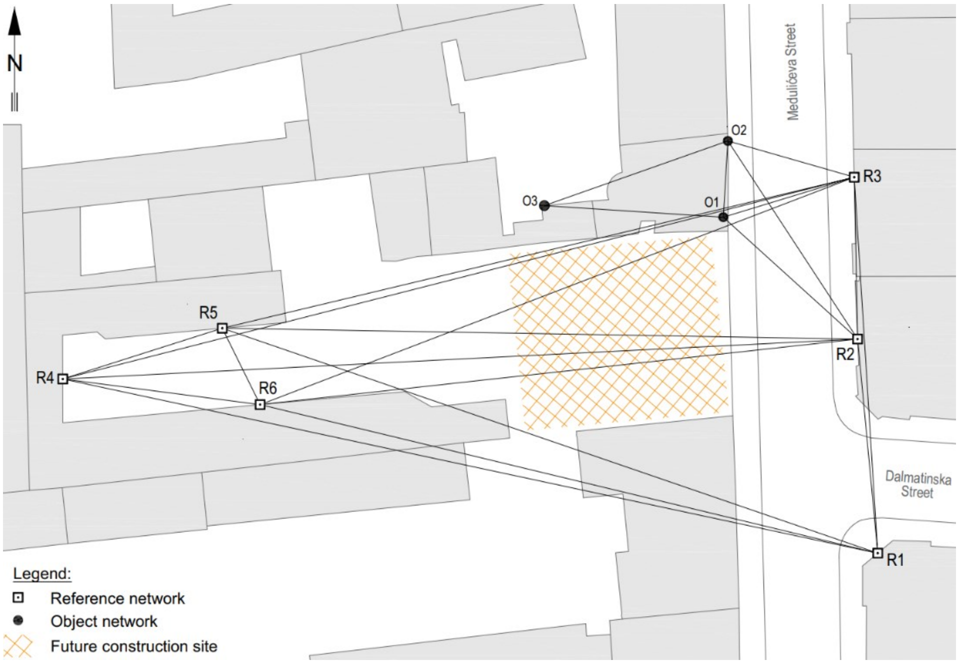

Figure 1. Established monitoring network surrounding the future construction site.

The monitoring network was designed as a classical 1D levelling network consisting of a reference network (R1 – R6) and an object network (O1 – O3). The present object network configuration is an initial configuration and incorporates only the northern adjacent building, but it will be expended in the following epochs to include the main building when its construction starts. All reference network points have been adequately stabilized by a chemical injection adhesive into the foundations of nearby buildings in such a way that they are outside deformation influences. Furthermore, all benchmarks have removable heads to secure them from destruction (Figure 2).

In May 2020, the stabilization of the network's benchmarks was performed and measurements of the zero epoch have been conducted. However, because of the recent seismic activity of the wider Zagreb area [10], a control measurement epoch of the established monitoring network has been performed in late November of 2020, before construction start. All geodetic measurements were conducted utilizing a precise digital level Leica DNA03 with a corresponding bar code invar levelling staff Leica GPCL2.

The instruments relevant technical specification can be found in the Leica DNA03 User Manual [12]. Prior to any field measurements, the instrument's measuring precision has been evaluated according to the international standard ISO 17123-2:2001 [13]. It was concluded that its measurement precision fulfils the declared specifications. In both epochs, the measurement procedure was uniform.

Measurement of height differences in the monitoring network was performed by the precise geometric levelling method whereby the BF (backsight – foresight) measurement sequence was applied. All network observations in the reference and control epoch are given in Table 1.

Table 1. Monitoring network observations in the reference and control epochs.

Levelling line		**Epoch 0 (reference)** *May 22, 2020*		**Epoch 1 (control)** *November 26, 2020*	
From	**To**	**Height diff. [m]**	**Distance [m]**	**Height diff. [m]**	**Distance [m]**
R1	R2	0.06696	27.4	0.06718	38.0
R1	R3	0.02623	47.1	0.02653	46.8
R1	R4	0.02294	93.5	0.02352	91.0
R1	R5	0.00728	82.6	0.00752	80.5
R1	R6	0.05175	73.3	0.05227	72.6
R2	R3	–0.04052	13.2	–0.04031	27.2
R2	R4	–0.04429	79.2	–0.04382	79.7
R2	R5	–0.05972	68.5	–0.05932	68.7
R2	R6	–0.01536	74.0	–0.01482	65.3
R3	R4	–0.00321	87.8	–0.00359	87.9
R3	R5	–0.01914	76.9	–0.01920	76.6
R3	R6	0.02546	69.2	0.02537	69.6
R4	R5	–0.01539	15.6	–0.01540	18.3
R4	R6	0.02906	19.5	0.02912	20.5
R5	R6	0.04448	15.0	0.04427	13.4
R2	O1	0.22681	18.1	0.22718	26.8
R2	O2	0.31892	24.5	0.31901	25.0
R3	O1	0.26750	23.4	0.26783	22.9
R3	O2	0.35947	19.7	0.35922	18.6
O1	O2	0.09209	10.6	0.09160	10.5
O1	O3	–0.44687	31.1	–0.44797	31.6
O2	O3	–0.53865	28.7	–0.53951	39.5

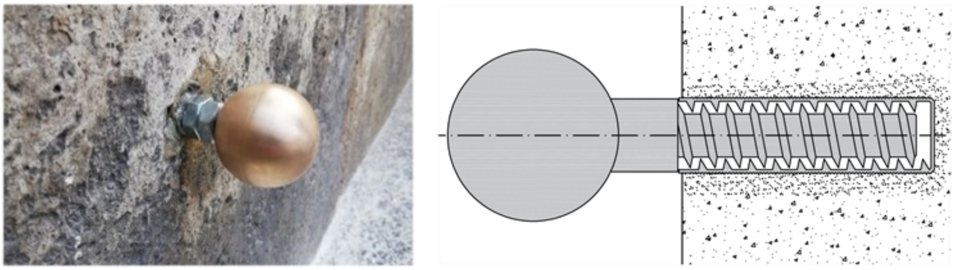

Figure 2. Benchmark stabilization type.

4. NETWORK ANALYSIS AND DISPLACEMENT DETERMINATION

According to the previously elaborated first step of the applied Karlsruhe deformation analysis method, individual least square adjustments of the monitoring network in the reference and control epochs, using the minimum trace datum, have been performed. The key results are shown in Table 2. Furthermore, based on these individual adjustment results, according to equation (1), the joint square form is determined and equal to $\Omega_0 = 1.426$ mm^2. According to the deformation analysis algorithm, the joint adjustment of both measurement epochs has been conducted and the corresponding square form calculated and amounts to $\Omega_J = 7.314$ mm^2.

Table 2. Key results of individual monitoring network adjustments.

Parameter	**Epoch 0** *May 22, 2020*	**Epoch 1** *November 26, 2020*
Degree of freedom b_i	14	14
Reference standard deviation $\overline{s}_i$ [mm]	0.20	0.25
Square form Ω_i [mm^2]	0.559	0.867

To test the stability of all network points the test statistic, according to equation (5), is calculated and amounts to $F = 14.444$. The critical value of *F* distribution for 8 and 28 degrees of freedom and the significance level $\alpha = 0.05$ is $F_{8,28,0.95} = 2.291$. Hence, it is concluded that unstable points exist in the monitoring network. In accordance with the described procedure of the Karlsruhe method, the localization od unstable points is conducted, and all key results are given in Table 3.

The localization of unstable network points has been performed in three iterations resulting in points O1, O3 and R1 being statistically determined as unstable. Within the third iteration, the calculated test statistics $F = 2.535$ is smaller than the critical value of *F* distribution $F_{5,28,0.95} = 2.558$, therefore no more unstable points exist in the network.

Table 3. Key results of unstable network point localization.

Network point	Ω_J [mm^2] *1. iteration*	Ω_J [mm^2] *2. iteration*	Ω_J [mm^2] *3. iteration*
R1	6.408	4.039	**1.640**
R2	6.098	3.403	1.684
R3	6.188	3.322	1.692
R4	6.534	3.509	1.766
R5	6.736	3.719	1.975
R6	6.593	3.573	1.829
O1	**3.327**	-	-
O2	4.655	2.769	2.128
O3	4.133	**2.439**	-
Stability testing results after unstable point exclusion			
Ω_J [mm^2]	4.293	2.549	2.072
Ω_h [mm^2]	2.866	1.122	0.646
f	7	6	5
F	8.037	3.672	2.535
$F_{f,b,0.95}$	2.359	2.445	2.558

Finally, based on the unstable network point localization, the reference benchmark R1 and the object benchmarks O1 and O3 have been statistically determined as unstable, i.e., have changed their position (height) between the control and reference epochs. A final network least square joint adjustment has been conducted to quantify the vertical displacements of the unstable points (Table 4) and to determine the final heights of the stable points (Table 5). In the adjustment procedure, the stable network points (R2, R3, R4, R5, R6 and O2) and the unstable points (O1, O3 and R1) are organized in the vector of unknown parameters $\mathbf{x}$ as stated in equation (3). According to the algorithm of the Karlsruhe method, equation (7), the significance of the determined vertical displacements is once again statistically tested. The test statistics (T_j) for each unstable point is greater than the critical value $F_{1,5,0.95} = 6.608$ which confirms the point's movement.

Table 4. Final deformation analysis results on unstable monitoring network points.

Network point	$\bar{H}_j'$ [m]	$\bar{H}_j''$ [m]	d_j [mm]	c_j [mm]	T_j	$F_{1,5,0.95}$
O1	100.2939	100.2943	0.4	0.20	21.928	6.608
O3	99.8471	99.8463	–0.8	0.32	36.464	6.608
R1	100.0001	99.9998	–0.3	0.27	9.358	6.608

Table 5. Final deformation analysis results on stable monitoring network points.

Network point	$\bar{H}_j$ [m]	c_j [mm]
R2	100.0670	0.09
R3	100.0265	0.09
R4	100.0229	0.13
R5	100.0075	0.12
R6	100.0519	0.12
O2	100.3859	0.10

The calculated vertical displacements of the monitoring network's unstable points with the corresponding 95%-confidence intervals (c_j) are graphically depicted on Figure 3. Furthermore, the 95%-confidence intervals of the adjusted heights of stable network points are shown.

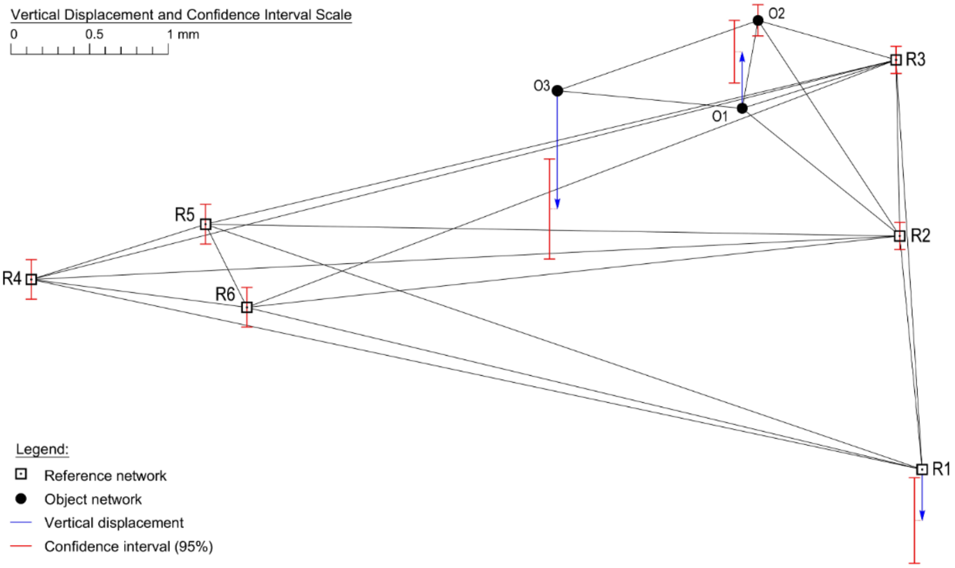

Figure 3. Monitoring network with vertical displacements and 95%-confidence intervals.

5. CONCLUSION

In this study the Karlsruhe deformation analysis method has been applied to test the stability of a 1D levelling monitoring network established for the purpose of vertical displacement determination of a five-story residential building during its construction. Measurements of the established monitoring network have been conducted in two epochs before the construction start, the reference epoch in May 2020 and the control epoch in November of 2020. According to the Karlsruhe method, the deformation analysis has been conducted and unstable points in the network have been detected. It is concluded, with a 95% probability, that the object points O1 and O3 and the reference point R1 have vertically moved between the two measurement epochs. The determined vertical movements of the unstable network points are below the one-millimeter level. By analyzing all relevant quality criteria, the network configuration, conducted measurements and adjustment results are evaluated as high quality and satisfactory. Although the magnitudes of the determined displacements are, from a civil engineering aspect, negligible, they give a vital information on the stability of the surrounding buildings and indicate that further measurements during construction are required.

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ANALIZA STABILNOSTI NIVELMANSKE MREŽE ZA POTREBE IZGRADNJE STAMBENE ZGRADE

Резиме: *Značaj geodetskih metoda za određivanje deformacija tokom izgradnje zgrada je dobro poznata tema. Razvijene su i aktivno se koriste mnoge klasične i savremene geodetske metode zasnovane na primeni mernih stanica, GNSS-a, laserskom skeniranju, fotogrametrijskim i drugim tehnologijama. Međutim, geometrijsko niveliranje je i dalje jedna od najtačnijih od svih geodetskih metoda merenja. Ovaj rad daje praktični primer primene analize deformacija na nivelmansku mrežu uspostavljenu za određivanje vertikalnih pomeranja tokom izgradnje stambene zgrade u Zagrebu u Republici Hrvatskoj. Pre početka izgradnje, stabilnost mreže je analizirana merenjem u dve epohe i primenom Karlsruhe metode analize deformacija. Predstavljeni su svi ključni rezultati.*

Кључне речи: *nivelmanska mreža, vertikalni pomaci, metoda Karlsruhe, stambena zgrada, izgradnja.*