COMPUTER SIMULATION 1D MODEL INDUCED OF TWO FREQUENT BY THE ACTION OF EXTERNAL DISPLACEMENT – PART 2

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Summary:. *In this paper is, imposed 1D dynami\c model with a resistance, of substrate excitation in domain of low frequency. Excitation with two different frequencies of displacement amplitudes ($\Omega_1 \neq \Omega_2 \neq 0$) are mathematically modeled and is a specificity of this research. Applying the Fourier transformations of the treated displacement amplitude in the frequency and time domain are showed that they are mapping proposed by the transfer function (I.M.Miličić, 2015). Constructed computer simulations confirmed displacement of structural systems at lower frequencies of external excitation controls stiffness.*

Keywords: *Simulation, dynamic model, FFT and IFFT algorithm, transfer function, displacement.*

1. INTRODUCTION

In research [4] and [2] we suggested particular solution for movement of 1D model which was depended by impost of external excitation. Solution for homogeneously part in differental equation for his model which is lose in time, only depend of external excitation.

We gave [4] a particular solution for forced damping oscillations with background resistance in the function of a dynamic coefficient that has practical application in the theoretical – experimental analysis of structures.

In previous research we implemented computer modeling with simulation for 1D dynamics model, which treated solution of movement who correspond with suggested transfer function "excitation – response". According to that solution movement of model is normalized by coefficient of disturbance and we have equation (12) in which the major factor for movement of 1D model was stiffness with low frequency of external excitation.

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Final objective of this research is theoretical treat one case of movement 1D model when external excitation is:

- imposed with two different frequency but with same displacement amplitudes – simulation of a quaetly dynamic effect,
- periodical function in domain with low but different frequency of amplitude displacement,
- imposed like two frequence and movement of 1D model is controled by stiffness of girder.

One more attention, our problem hasn't been finished by this research because the border of the low – incidence area of the excitation on the right hasn't been determined yet.

2. COMPUTER SIMULATION

The input data and calculating the natural frequencies and damping

$$\omega := \sqrt{\frac{c}{m}} \qquad \omega = 12.5 \qquad f := \frac{\omega}{2\cdot\pi} \qquad f = 1.99$$

$$b := 2\cdot m\cdot\omega\cdot\xi \qquad b = 1.6\times10^{3} \qquad \omega_d := \omega\cdot\sqrt{\left(1-\xi^2\right)} \qquad \omega_d = 12.44$$

$$f_d := \frac{\omega_d}{2\cdot\pi} \qquad T_d := \frac{1}{f_d} \qquad T_d = 0.505 \qquad \frac{T_d}{10} = 0.0505$$

Two frequency excitation input,

$$A_1 := 5 \qquad A_2 := 5$$

$$\Omega_1 := \frac{0.03}{10}\cdot\omega \qquad \Omega_2 := \frac{0.9}{10}\cdot\omega$$

excitation model:

$$\Delta_i := A_1\cdot\cos\left(\Omega_1\cdot t_i\right) + A_2\cdot\sin\left(\Omega_2\cdot t_i\right)$$

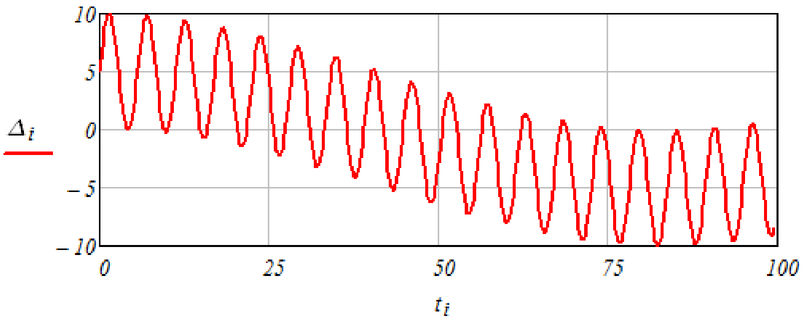


Figure 1 – Two frequency excitation with a retention time t=100s

The general form of the equation of the movement of the model,

$$x(t) = A_i \cdot P(\psi_i) \cdot cos(\Omega_i \cdot t + \theta_i)$$

superposition of individual responses

$$x(t) = X_1 \cdot cos(\Omega_1 \cdot t + \theta_1) + X_2 \cdot sin(\Omega_2 \cdot t + \theta_2)$$

Where is:

- amplitude response $X_i = \frac{c}{c} \cdot A_i \cdot P(\psi_i)$
- coefficient of disorder $\psi_i = \frac{\Omega_i}{\omega} \quad i = 1, 2$

Amplitude and phase angles response model,

amplitude: $P(\xi, \psi) := \frac{1}{\sqrt{(1-\psi^2)^2 + (2 \cdot \xi \cdot \psi)^2}}$ scaling factor: $\lambda := \frac{1}{A_2}$

phase angle:

$$\theta(\xi, \psi) := \begin{array}{|l} \theta \leftarrow -atan\left(\frac{2 \cdot \xi \cdot \psi}{1-\psi^2}\right) \\ \theta \leftarrow \theta - \pi \;\; if \;\; \psi > 1 \\ return \;\; \theta \end{array}$$

$$\theta(\xi, \psi) := -\theta(\xi, \psi) \cdot \frac{180}{\pi}$$

First response:

$\psi_1 := \frac{\Omega_1}{\omega}$ $\quad \psi_1 = 0.003$

$P(\psi_1) = 1$ $\quad \theta_1 := \theta(\psi_1)$

$X_1 := \frac{c}{c} \cdot A_1 \cdot P(\psi_1)$ $\quad X_1 = 5.0000$ $\quad \theta_1 \cdot \frac{180}{\pi} = -0.0344$

Second response:

$\psi_2 := \frac{\Omega_2}{\omega}$ $\quad \psi_2 = 0.09$

$P(\psi_2) = 1.008$ $\quad \theta_2 := \theta(\psi_2)$

$X_2 := A_2 \cdot P(\psi_2)$ $\quad X_2 = 5.0400$ $\quad \theta_2 \cdot \frac{180}{\pi} = -1.0396$

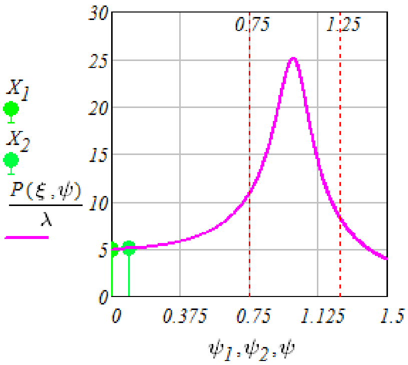


Figure 2 – Amplitude response

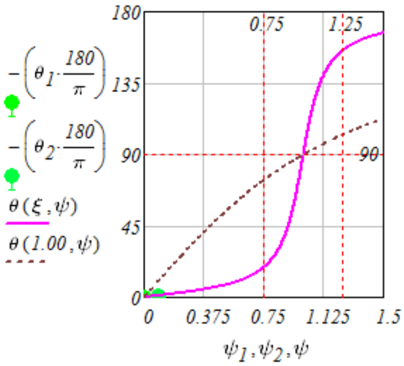


Figure 3 – The phase angles response

The general form of the equation of motion model – response:

$$x_i := X_1 \cdot cos\left(\Omega_1 \cdot t_i + \theta_1\right) + X_2 \cdot sin\left(\Omega_2 \cdot t_i + \theta_2\right)$$

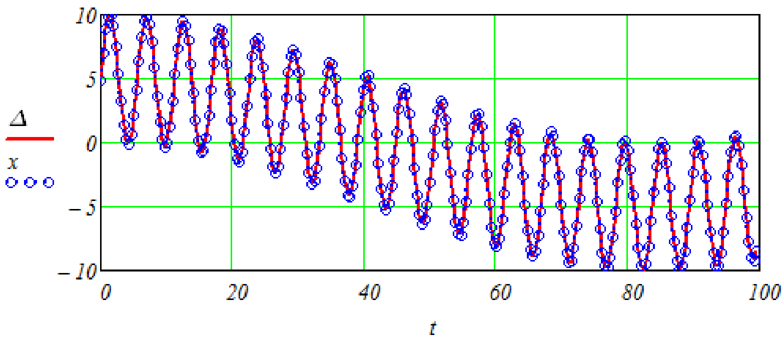


Figure 4 – Displacement 1D dynamic model for t=100s (excitation – response)

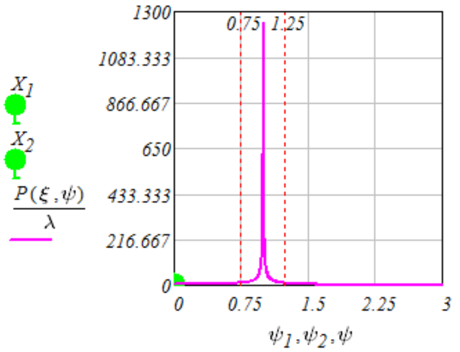
Comment: Based on one part of simulation, we are noticed oscillatory movement of model in time domain with compatibile values of movement amplitudes of displacement. We are monitor variety of stiffness (column 1) simulated by constant coefficient of damping (column 2; table 1, 2 and 3). In equation (9) at [4], we calculated amplitudes of respons by frequency of excitation (column 3 and 4) and their relationship between natural and forced (column 7).

In columns (5) and (6) we calculated particular, where:

- they are different between each other *($\Omega_1 \neq \Omega_2 \neq 0$)*,
- are exist in function of stiffness of girder.

However, in this case we have notice influence of the coefficient of damping but it doesn't important for finally moving of system. In that way we leave the possibility for quantitative determination of theoretical right side of domain where stiffness has dominant part in control point of disceplement on girder for quaetly dynamics load.

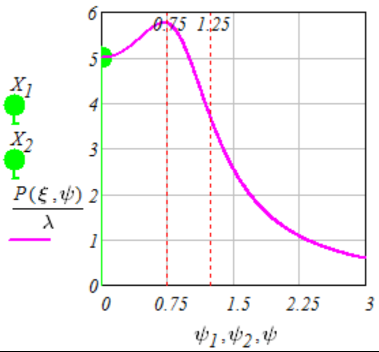
Table 1 – The first case



c (N/m)	ξ (-)	X_1 (mm)	X_2 (mm)	Ω_1 (1/s)	Ω_2 (1/s)	f/ω_d (-)
1	2	3	4	5	6	7
10^2				1.186e-3	0.036	
10^4				0.012	0.356	
10^5	0.0	5.0000	5.0408	0.038	1.125	0.159
10^7				0.375	11.25	
10^8				1.186	35.576	

Comment: Noticed the oscillatory movement of model with two amplitudes with model response.

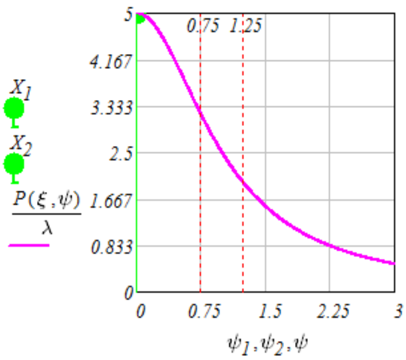
Table 2 – The second case



c (N/m)	ξ (-)	X_1 (mm)	X_2 (mm)	Ω_1 (1/s)	Ω_2 (1/s)	f/ω_d (-)
1	2	3	4	5	6	7
10^2				1.186e-3	0.036	
10^4				0.012	0.356	
10^5	0.5	5.0000	5.0202	0.038	1.125	0.184
10^7				0.375	11.25	
10^8				1.186	35.576	

Comment: We noticed partially oscillatory movement with two amplitudes model response and coefficient damping of system by 50 %.

Table 3 – The third case



c (N/m)	ξ (-)	X_1 (mm)	X_2 (mm)	Ω_1 (1/s)	Ω_2 (1/s)	f/ω_d (-)
1	2	3	4	5	6	7
10^2				1.186e-3	0.036	
10^4				0.012	0.356	
10^5	1.0	5.0000	4.9598	0.038	1.125	∞
10^7				0.375	11.25	
10^8				1.186	35.576	

Comment: We noticed border case where isn't exist oscillatory movement because coefficient damping of system was 100 %.

Accordingly, priority is consideration first case in which we have oscillatory movement 1D dynamics model. We are changing coefficient damping of system for constant stiffness (Table 4) and we have values of amplitudes which corespond with transfer function "excitation – response".

Table 4 – Results of simulation for one system stiffness value

C	**ξ**	$\mathbf{X_1}$	$\mathbf{X_2}$	$\mathbf{\Omega_1}$	$\mathbf{\Omega_2}$	$\mathbf{f/\omega_d}$
(N/m)	(-)	(mm)	(mm)	(1/s)	(1/s)	(-)
	0.0		5.0408			0.159
	0.1		5.0400			0.160
	0.2		5.0375			0.162
	0.3		5.0334			0.167
	0.4		5.0276			0.174
10^5	0.5	5.000	5.0202	0.038	1.125	0.184
	0.6		5.0112			0.199
	0.7		5.0006			0.223
	0.8		4.9885			0.265
	0.9		4.9749			0.365
	1.0		4.9598			∞

We will first calculated three model response with two amplitudes of external excitation with the same frequency in function of changing coefficient of damping.
We noticed that for different values of coefficient damping of system, amplitude of model response are the same value. So, movement of 1D model control exclusively stiffness of girder.
We consider how values amplitude of external excitation and which solution we have for model movement accordingly equation (9) in [4] ?

Where:

- $X^{\langle 0 \rangle}$ – coefficient of system damping
- $X^{\langle 1 \rangle}$ – first amplitude of moving excitation
- $X^{\langle 2 \rangle}$ – first amplitude of moving response
- $X^{\langle 3 \rangle}$ – second amplitude of moving excitation
- $X^{\langle 4 \rangle}$ – second amplitude of moving response
- $X^{\langle 5 \rangle}$ – third amplitude of moving excitation
- $X^{\langle 6 \rangle}$ – third amplitude of moving reponse

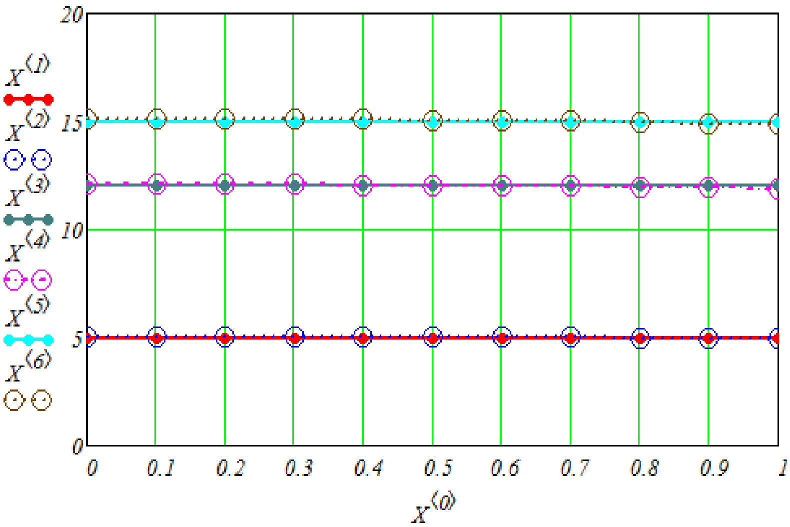


Figure 5 – Graphic representation of simulation results displacement 1D model (excitation – response)

Comment: Values amplitude of moving excitation in 1D dynamic model isn't depend because movement of system is according to transfer function have same values amplitude of movement external excitation by response for each case.
However, if we consider particular this three simulation of 1D model, we have results which we will present on Fig. 6, 7 and 8.

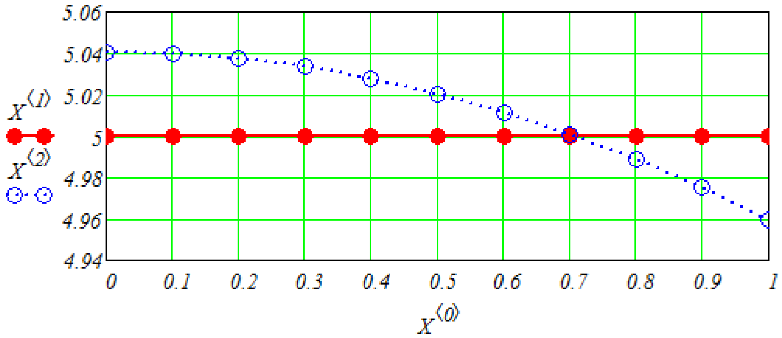


Figure 6 – First simulation – graphic representation of simulation results displacement 1D model (excitation – response)

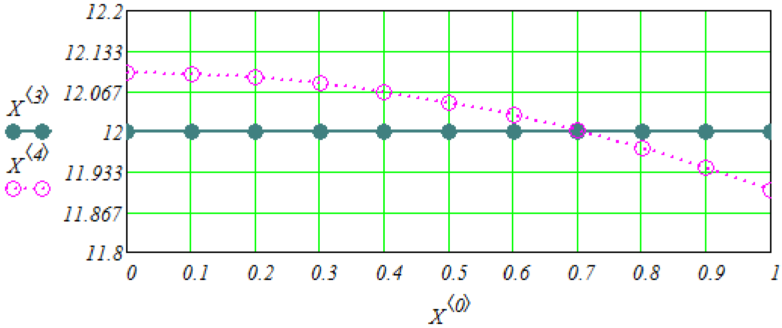


Figure 7 – Second simulation *– graphic representation of simulation results displacement 1D model (excitation – response)*

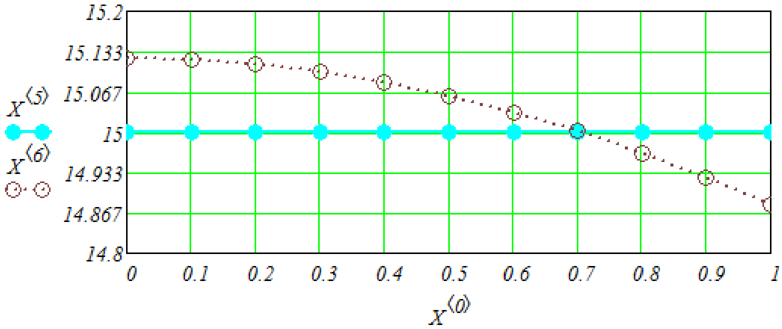


Figure 8 – Third simulation – graphic representation of simulation results displacement 1D model (excitation – response)

3. RECONSTRUCTION RESPONSE 1D MODELS

3.1. FFT transformation

- **Time domain**
- **Frequency domain**

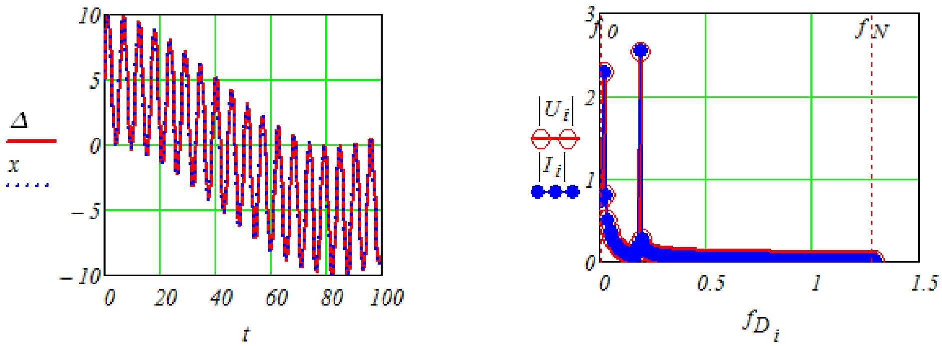


Figure 9 – Displacement 1D model (excitation-response) for t=100s

Excitation *Response*

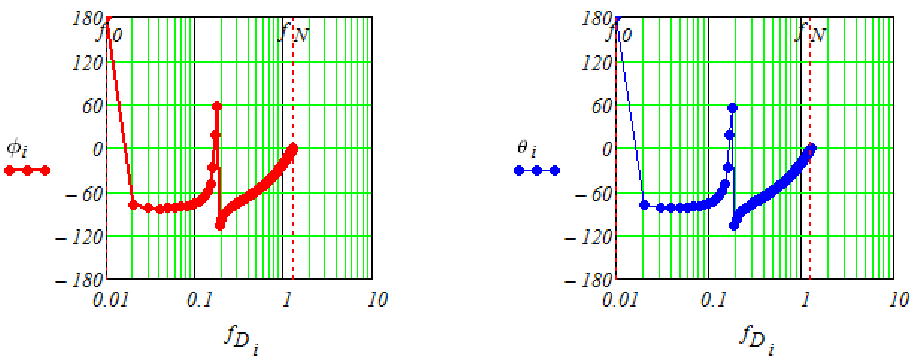


Figure 10 – The range of phase angle for t=100s

4. CONCLUSION

Based on computer simulation in fact imposed of two frequent extitation *($\Omega_1 \neq \Omega_2 \neq 0$)*, we have:

- for *($\Omega_1 \neq \Omega_2 \neq 0$)* system response with no significant deviation,
- quantitative model response which have statics character who is controled by stiffness of girder,
- precise transfer function “response – excitation” using FFT and IFFT with the Furie’s transformations,
- ability to determine with simulations the unknown right border of the area which was controlled by stiffness in fact of quaetly dynamic load.

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РАЧУНАРСКА СИМУЛАЦИЈА 1Д МОДЕЛА ПОБУЂЕНОГ ДВО ФРЕКВЕНТНИМ ДЕЈСТВОМ СПОЉАШЊИХ ПОМЕРАЊА – ДЕО 2

***Резиме**: У овом раду наметнута је, 1Д динамичком моделу са отпором подлоге, побуда у подручју ниских учестаности. Математички је моделирана побуда са две различите учестаности амплитуда померања ($\Omega_1 \neq \Omega_2 \neq 0$) и представља посебност овог истраживања. Примењујући алгоритме Фуријеових трансформација третиране амплитуде померања у фреквентном и временском домену респектују пресликавање предложено функцијом преноса (И.М.Миличић, 2015). Спроведеним рачунарским симулацијама потврђено је да померања конструктивних система при нижим учестаностима спољашње побуде контролише крутост.*

***Кључне речи:** Симулација, динамички модел, FFT и IFFT алгоритам, функција преноса, померања.*