ANALYSIS OF PLATES BY THE SECOND ORDER THEORY NUMERICAL METHODS

Besim Demirović [1]
Mirela Garić-Demirović [2]
Nedim Osmić [3]

UDK: 624.042.072.1:519.6
DOI: 10.14415/konferencijaGFS2018.013

Summary: *The paper presents an analysis of the bending of plates loaded vertically and in the direction of the middle plane. The procedure was done numerically using the final difference method. The calculation of the displacement of the panel at the points of discretization was carried out by iterative methods taking into account the second-order effects. By analyzing the plate element in a deformed position, the differential bending equation of the plate was implemented, in which the contribution of the transient forces to the bending and deformation moments is introduced. The methods of calculation shown are modeled by the geometric nonlinearity of the panels. Through numerical examples, the calculation procedure was presented and the results analysis was performed.*

Keywords: *geometric nonlinearity, second order theory, finite difference method.*

1. INTRODUCTION

In the case of thin plates strained at the same time on bending and loading in the plane, there is an increase in deformations due to normal and shifting forces.
By analyzing the deformed plate element, the external load affects the change of transient forces and deformations. Load in the flat panel causes additional static influences and deformations called second-order effects.
Of all the influences of the second order, the moments of bending and deforestation of the board have a significant role, and in this paper are analyzed.
The bending moments obtained by the second-order theory in some cases of loading of the panels can not be ignored.
The bending moments in this paper were obtained iteratively with gradual approximation to the equilibrium state of the plate.

[1] Dr sci. Besim Demirović, Structural Engineer, University of Tuzla, Faculty of Mining, Geology and Civil Engineering, Univerzitetska 2, 75000 Tuzla, Bosnia and Herzegovina, e – mail: besim_demirovic@yahoo.com
[2] Dr sci. Mirela Garić-Demirović, University of Tuzla, Faculty of Natural Sciences and Mathematics, Univerzitetska 4, 75000 Tuzla, Bosnia and Herzegovina.
[3] Nedim Osmić, Ma. Civil Engineering, University of Tuzla, Faculty of Mining, Geology and Civil Engineering, Univerzitetska 2, 75000 Tuzla, Bosnia and Herzegovina.

2. ANALYSIS OF PLATE ELEMENTS BY THE SECOND ORDER THEORY

The analysis of the panels according to the second-order theory is justified in cases where significant normal forces occur in slender plates. The plate is loaded with normal and tangential load in relation to the middle plane. The load combinations perpendicular and in the direction of the middle plate are common, as in the case of walls and ceilings. The budget assumes that the loading of the board before and after deformation remains the same intensity and direction. The elementary part of the dimensions dx and dy is separated from the flat isotropic plate of thickness h, on which the external and internal forces are shown, Fig. 1.a.

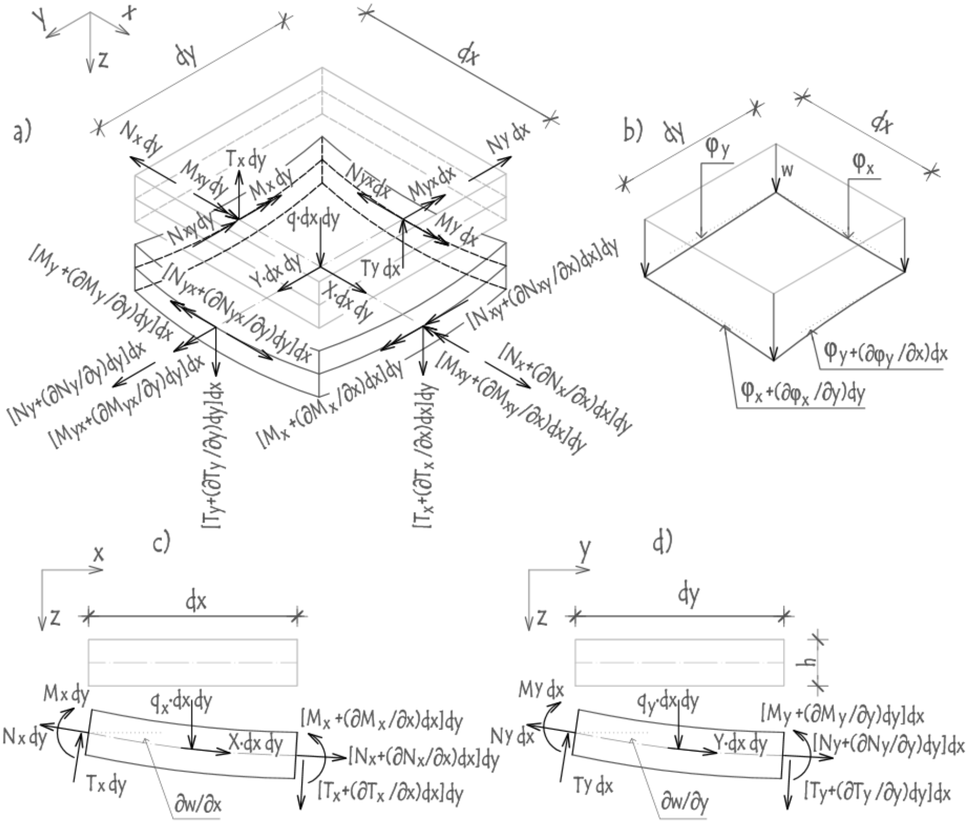

Figure 1. a) The cross member forces of the dx · dy panel, b) the deformation angles of the middle plane c) the cross-section of the element at the zx plane, d) the cross-section of the element in the plane zy

The equilibrium conditions of the elementary part of the plate are established on the deformed element. In the deformation of the middle plane, the intersection forces N and

T close the angles $\varphi_x = \partial w / \partial x$ and $\varphi_y = \partial w / \partial y$, Fig. 1.c. and d. In the analysis of the panel element, the following assumptions are introduced:

- Cross sections are administratively on the middle plane of the board before and after deformation;
- The middle plane of the plate after deformation remains unchanged;
- The stresses σz, τ_{xz} and τ_{yz} are ignored;
- The material is linearly elastic, homogeneous and isotropic;
- For deformation quantities it is valid: $(\varepsilon_x, \varepsilon_y, \tau_{xy}, \varphi_x, \varphi_y) << 1$;
- The increase in the load plate from the load is $\Delta w \neq 0$
- The intermittent forces N_x, N_y, N_{xy} are determined on an undeformed plate (first-order theory).

According to the adopted assumption of the determination of force N on an unadjusted plate, sufficient accuracy of the solution is achieved. The theory of the analysis of a deformed plate element in which the transient forces N_x, N_y, N_{xy} are determined on an undefined configuration, is called linearized second order theory. From the conditions of equilibrium of all forces with axis we obtain:

$$\begin{aligned}
&-T_x dy + T_x dy + \frac{\partial T_x}{\partial x} dx\,dy - T_y dx + T_y dx + \frac{\partial T_y}{\partial y} dy\,dx - \\
&-N_x dy \frac{\partial w}{\partial x} + N_x dy\left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2} dx\right) + \overbrace{\frac{\partial N_x}{\partial x} dx\,dy\left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2} dx\right)}^{\approx 0} - \\
&-N_y dx \frac{\partial w}{\partial y} + N_y dx\left(\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2} dy\right) + \overbrace{\frac{\partial N_y}{\partial y} dy\,dx\left(\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2} dy\right)}^{\approx 0} - \\
&-N_{yx} dx \frac{\partial w}{\partial x} + N_{yx} dx \frac{\partial w}{\partial x} + N_{yx} dx \frac{\partial \varphi_x}{\partial y} dy + \overbrace{\frac{\partial N_{yx}}{\partial y} dy\,dx\left(\varphi_x + \frac{\partial \varphi_x}{\partial y} dy\right)}^{\approx 0} - \\
&-N_{xy} dy \frac{\partial w}{\partial y} + N_{xy} dy \frac{\partial w}{\partial y} + N_{xy} dy \frac{\partial \varphi_y}{\partial x} dx + \overbrace{\frac{\partial N_{xy}}{\partial x} dx\,dy\left(\varphi_y + \frac{\partial \varphi_y}{\partial x} dx\right)}^{\approx 0} + \\
&+q \cdot dx\,dy + X \cdot dx\,dy \frac{\partial w}{\partial x} + Y \cdot dx\,dy \frac{\partial w}{\partial y} = 0
\end{aligned} \tag{1}$$

In equation (1) projections of transverse forces on z axis are approximately equal to T_x and T_y forces, since $\cos \varphi x \approx 1$ and $\cos \varphi_y \approx 1$. If the higher order elements are ignored, adopt the $N_{yx} = N_{xy}$ connection and the expression is divided by $dx \cdot dy$, after sorting the equations (1) of the form:

$$\frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial}{\partial x}\frac{\partial w}{\partial y} + q + X\frac{\partial w}{\partial x} + Y\frac{\partial w}{\partial y} = 0 \tag{2}$$

The links between transverse forces and the displacement of the middle plane are in the form:

$$T_x = -K\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x\,\partial y^2}\right); \qquad T_y = -K\left(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2\,\partial y}\right) \tag{3}$$

By replacing the transversal forces in the expression (2) with the partial displacement of the plate, the following is given:

$$\begin{aligned} &-K\frac{\partial}{\partial x}\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x\,\partial y^2}\right) - K\frac{\partial}{\partial y}\left(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2\,\partial y}\right) + \\ &+N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y} + N_y\frac{\partial^2 w}{\partial y^2} + q + X\frac{\partial w}{\partial x} + Y\frac{\partial w}{\partial y} = 0 \end{aligned} \tag{4}$$

The bending equation of the panel (4) after the arrangement of the partial derivatives can be written in the form (5):

$$-K\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = -\left(q + N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y} + N_y\frac{\partial^2 w}{\partial y^2} + X\frac{\partial w}{\partial x} + Y\frac{\partial w}{\partial y}\right) \tag{5}$$

After multiplying with (-1 / K), the bending equation of a plate according to linearized second order theory is of the form:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{K}\left(q \pm N_x\frac{\partial^2 w}{\partial x^2} \pm 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y} \pm N_y\frac{\partial^2 w}{\partial y^2} + X\frac{\partial w}{\partial x} + Y\frac{\partial w}{\partial y}\right) \tag{6}$$

In the equation (6) the displacements of the deformed plate of the plate are unknown. The X and Y forces are known and act as surface tangential forces in the middle plane of the plate. If these forces are equal to zero, their members are omitted in the equation (6). According to the conventions (Figure 1.a), the sign (-) in front of the transient forces Nx, Ny, Nxy is valid when the force values are positive, and the sign (+) is used when the negative forces are N. If we assume that the load X and Y u plane plates equals zero, according to the adopted sign of the force N of the equation (6) is in the form:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{K}\left(q \pm N_x\frac{\partial^2 w}{\partial x^2} \pm 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y} \pm N_y\frac{\partial^2 w}{\partial y^2}\right) \tag{7}$$

3. APPLICATION OF THE FINITE DIFFERENCE METHOD

The Finite Difference Method (FDM) represents the numerical procedure for solving the differential equation in the node. We divide the board into fewer elements and we seek the differential equation solution for the individual nodes of the division. In the vicinity of the point k, in which we seek the solution of the differential equation (7), we make a division by x axis into parts of length Δx, and along the y axis Δy, Fig. 2.

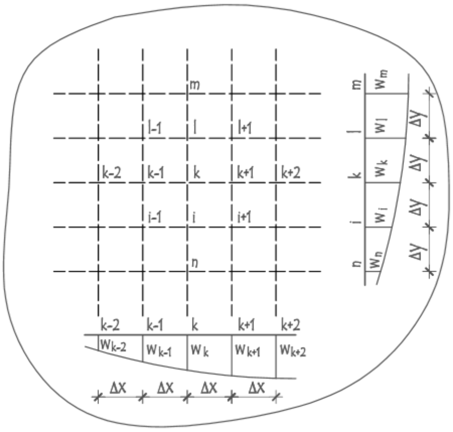

Figure 2. Discretization of the plate

The functions of moving the plate along the x and y axes obtained by the method of finite differences depend on the angle of the points shown in Figure 2. According to the literature [x], the term for the left side is known equations (7) by the finite difference method. The expressions for the curvature of the plate located on the right side of the equation (7) with the forces N_x, N_y and N_{xy} are the form:

$$\left(\frac{\partial^2 w}{\partial x^2}\right)_k \approx \frac{w_{k-1}-2w_k+w_{k+1}}{\Delta x^2}; \qquad \left(\frac{\partial^2 w}{\partial y^2}\right)_k \approx \frac{w_l-2w_k+w_i}{\Delta y^2};$$
$$\left(\frac{\partial^2 w}{\partial x \partial y}\right)_k \approx \frac{w_{l+1}-w_{l-1}-w_{i+1}+w_{i-1}}{4\Delta x \Delta y} \tag{8}$$

By replacing the fourth and second degree derivatives with the bending equation of the plate (7), we obtain:

$$w_k\left[6\left(\alpha^2+\frac{1}{\alpha^2}\right)+8\right]-4\left[\left(1+\alpha^2\right)\left(w_{k+1}+w_{k-1}\right)+\left(1+\frac{1}{\alpha^2}\right)\left(w_l+w_i\right)\right]+$$
$$+2\left(w_{l+1}+w_{l-1}+w_{i+1}+w_{i-1}\right)+\alpha^2\left(w_{k+2}+w_{k-2}\right)+\frac{1}{\alpha^2}\left(w_m+w_n\right)=$$
$$=\frac{\Delta x^2}{K}\Big[q_k\,\alpha^2\Delta x^2 \pm N_x\,\alpha^2\left(w_{k-1}-2w_k+w_{k+1}\right)\pm$$
$$\pm N_{xy}\,\frac{\alpha}{2}\left(w_{l+1}-w_{l-1}-w_{i+1}+w_{i-1}\right)\pm N_y\left(w_l-2w_k+w_i\right)\Big] \tag{9}$$

where is $\alpha=\frac{\Delta y}{\Delta x}$, $K=\frac{E\cdot h^3}{12\left(1-\nu^2\right)}$. Za $\Delta y=\Delta x$ For the expression of the equation (9), the form is:

$$20w_k - 8\left(w_{k+1} + w_{k-1} + w_l + w_i\right) + 2\left(w_{l+1} + w_{l-1} + w_{i+1} + w_{i-1}\right) +$$
$$+\left(w_{k+2} + w_{k-2} + w_m + w_n\right) = \frac{\Delta x^2}{K}\Big[q_k\,\Delta x^2 \pm N_x\left(w_{k-1} - 2w_k + w_{k+1}\right) \pm \qquad (10)$$
$$\pm N_{xy}\frac{1}{2}\left(w_{l+1} - w_{l-1} - w_{i+1} + w_{i-1}\right) \pm N_y\left(w_l - 2w_k + w_i\right)\Big]$$

The equation (10) is solved iteratively by the fact that in the i-th iterative step of moving the nodes on the right side of the equation, they are taken from the i-1 iterative step. In the i-th iterative step the equation is of the formform:

$$20w_k^i - 8\left(w_{k+1}^i + w_{k-1}^i + w_l^i + w_i^i\right) + 2\left(w_{l+1}^i + w_{l-1}^i + w_{i+1}^i + w_{i-1}^i\right) +$$
$$+\left(w_{k+2}^i + w_{k-2}^i + w_m^i + w_n^i\right) = \frac{\Delta x^2}{K}\Big[q_k\,\Delta x^2 \pm N_x\left(w_{k-1}^{i-1} - 2w_k^{i-1} + w_{k+1}^{i-1}\right) \pm \qquad (11)$$
$$\pm N_{xy}\frac{1}{2}\left(w_{l+1}^{i-1} - w_{l-1}^{i-1} - w_{i+1}^{i-1} + w_{i-1}^{i-1}\right) \pm N_y\left(w_l^{i-1} - 2w_k^{i-1} + w_i^{i-1}\right)\Big]$$

In the matrix form, the system of n equations can be written in the form:

$$[A]\cdot[w]^i = [B]^{i-1}$$
(12)

where is: [A] -matrix of members with unknown plate displacement, [w] -matrix of the displacement column of the board and the [B] -matrix of the columns of free members which depend on the load, the force N and the mechanical characteristic of the panel.

4. EXAMPLE

In the example, the second-order effects on the reinforced concrete wall are loaded with load p perpendicular to the middle plane and vertical load at the top of the wall intensity q = 300 kN / m. In the budget, the geometric imperfection of the panel according to the expression:

$w_0 = f_0 \sin\frac{\pi x}{a}\cdot\sin\frac{\pi y}{b}$, where f_0=0,7b/300=0,0093 m.
d=15 cm; μ=0,2 ; E=3x10^7 kN/m^2; K=8789,06 kNm^2/m.

The unknown movements of the panel are: w_8, w_9, w_{10}, w_{11}, w_{12}, w_{14}, w_{15}, w_{16}, w_{17}, w_{18}, w_{20}, w_{21}, w_{22}, w_{23}, w_{24}, w_{26}, w_{27}, w_{28}, w_{29}, w_{30}.

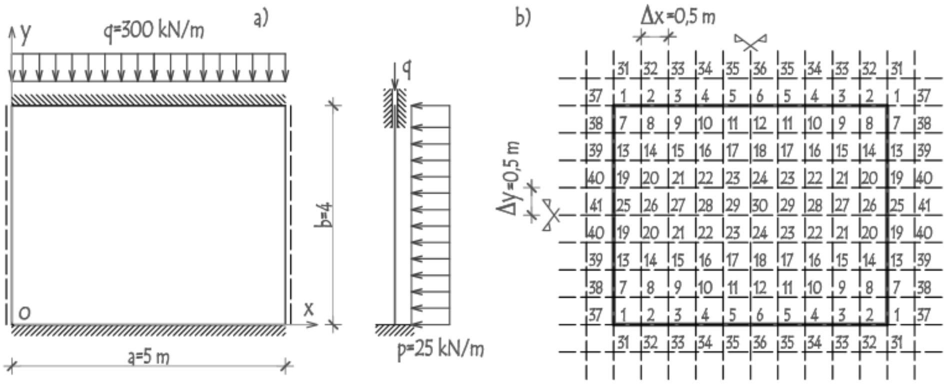

Figure 3. a) Disposition of the wall, b) wall discretization on elements 0.5 m long

The forces due to wall load in all internal nodes are: N_x = -60 kN and N_y = -300 kN. Moving supports supported along the side of the wall b with the sideways side displacement, which is why the smashing forces are N_{xy} = 0 in all nodes.
The change of free matrix members [B] through iteration is:

$$[B]^{(1)T} = [0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778 \quad 0.0001778]$$

$$[B]^{(2)T} = [0.0001791 \quad 0.0001796 \quad 0.0001804 \quad 0.0001807 \quad 0.000181 \quad 0.0001812 \quad 0.0001846 \quad 0.0001867 \quad 0.0001882 \quad 0.000195 \quad 0.000183 \quad 0.0001872 \quad 0.0001906 \quad 0.0001928 \quad 0.0001934 \quad 0.0001832 \quad 0.0001883 \quad 0.0001919 \quad 0.0001942 \quad 0.0001951]$$

$$[B]^{(3)T} = [0.000179 \quad 0.0001795 \quad 0.0001803 \quad 0.0001806 \quad 0.0001808 \quad 0.0001812 \quad 0.0001846 \quad 0.0001868 \quad 0.0001882 \quad 0.000195 \quad 0.0001831 \quad 0.0001873 \quad 0.0001908 \quad 0.000193 \quad 0.0001936 \quad 0.0001833 \quad 0.0001885 \quad 0.0001921 \quad 0.0001944 \quad 0.0001954]$$

The matrix of the members with the unknown movement of the board is in the form:

$$[A]=\begin{bmatrix}
20, -8, 1, 0, 0, -8, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, \\
-8, 21, -8, 1, 0, 2, -8, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, \\
1, -8, 21, -8, 1, 0, 2, -8, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, \\
0, 1, -8, 22, -8, 0, 0, 2, -8, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, \\
0, 0, 2, -16, 21, 0, 0, 0, 4, -8, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, \\
-8, 2, 0, 0, 0, 19, -8, 1, 0, 0, -8, 2, 0, 0, 0, 1, 0, 0, 0, 0, \\
2, -8, 2, 0, 0, -8, 20, -8, 1, 0, 2, -8, 2, 0, 0, 0, 1, 0, 0, 0, \\
0, 2, -8, 2, 0, 1, -8, 20, -8, 1, 0, 2, -8, 2, 0, 0, 0, 1, 0, 0, \\
0, 0, 2, -8, 2, 0, 1, -8, 21, -8, 0, 0, 2, -8, 2, 0, 0, 0, 1, 0, \\
0, 0, 0, 4, -8, 0, 0, 2, -16, 20, 0, 0, 0, 4, -8, 0, 0, 0, 0, 1, \\
1, 0, 0, 0, 0, -8, 2, 0, 0, 0, 20, -8, 1, 0, 0, -8, 2, 0, 0, 0, \\
0, 1, 0, 0, 0, 2, -8, 2, 0, 0, -8, 21, -8, 1, 0, 2, -8, 2, 0, 0, \\
0, 0, 1, 0, 0, 0, 2, -8, 2, 0, 1, -8, 21, -8, 1, 0, 2, -8, 2, 0, \\
0, 0, 0, 1, 0, 0, 0, 2, -8, 2, 0, 1, -8, 22, -8, 0, 0, 2, -8, 2, \\
0, 0, 0, 0, 1, 0, 0, 0, 4, -8, 0, 0, 2, -16, 21, 0, 0, 0, 4, -8, \\
0, 0, 0, 0, 0, 2, 0, 0, 0, 0, -16, 4, 0, 0, 0, 19, -8, 1, 0, 0, \\
0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 4, -16, 4, 0, 0, -8, 20, -8, 1, 0, \\
0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 4, -16, 4, 0, 1, -8, 20, -8, 1, \\
0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 4, -16, 4, 0, 1, -8, 21, -8, \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 8, -16, 0, 0, 2, -16, 20,
\end{bmatrix}$$

Figure 4 shows the diagrams of the damping of the panel and the bending moments obtained by the theory of the first and second order.

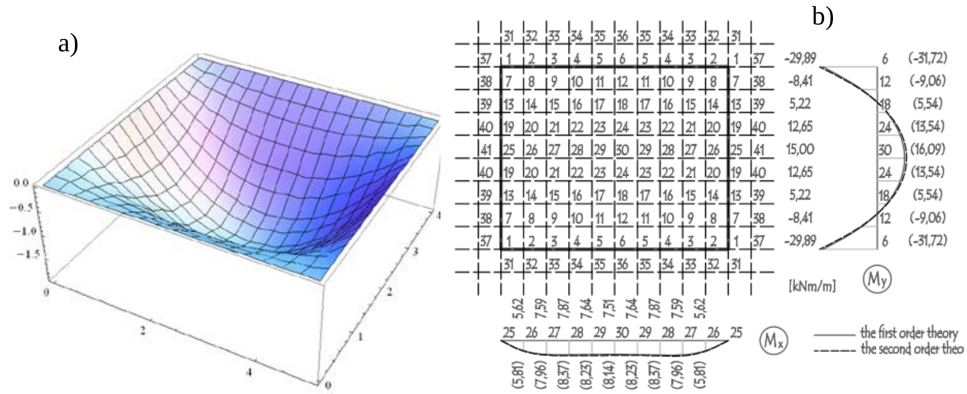

Figure 4. a) Plate surface [mm], b) Bending moments [kNm / m]

The results of the calculations of the deflection and the moment of the bending of the wall are shown in Table 1.

Table 1. Results of the calculation of the wall

Node	8	9	10	11	12	14	15	16	17	18
w_0 [mm]	1,110	2,090	2,880	3,380	3,560	2,030	3,870	5,320	6,250	6,580
Iteration	Wiring angles of the wall w [mm]									
1. it.	0,168	0,292	0,370	0,412	0,425	0,409	0,727	0,939	1,057	1,095
2. it.	0,176	0,307	0,391	0,436	0,451	0,431	0,768	0,995	1,124	1,165
3. it.	0,176	0,307	0,391	0,437	0,451	0,431	0,768	0,996	1,124	1,166
Bending moments [kNm/m]										
Mx(I)	1,08	0,57	-0,13	-0,62	-0,79	3,62	4,38	4,13	3,74	2,39
Mx(II)	1,06	0,54	-0,17	-0,67	-0,83	3,71	4,55	4,36	4,02	2,66
ΔMx [%]	-1,23	-5,18	22,94	6,53	4,17	2,41	3,69	5,25	6,82	10,26
My(I)	-2,24	-4,74	-6,75	-8,00	-8,41	2,61	4,09	4,82	5,13	5,22
My(II)	-2,44	-5,12	-7,28	-8,61	-9,06	2,70	4,26	5,06	5,42	5,54
ΔMy [%]	8,17	7,44	7,20	7,16	7,17	3,31	4,03	4,70	5,32	5,88
Node	20	21	22	23	24	26	27	28	29	30
w_0 [mm]	2,660	5,050	6,950	8,170	8,590	2,870	5,470	7,520	8,840	9,300
Iteration	Wiring angles of the wall w [mm]									
1. it.	0,595	1,067	1,390	1,572	1,631	0,663	1,193	1,557	1,764	1,831
2. it.	0,628	1,130	1,476	1,674	1,738	0,700	1,264	1,655	1,880	1,952
3. it.	0,628	1,131	1,477	1,675	1,739	0,700	1,265	1,656	1,881	1,954
Bending moments [kNm/m]										
Mx(I)	5,12	6,78	6,91	6,63	6,49	5,62	7,59	7,87	7,64	7,51
Mx(II)	5,29	7,09	7,34	7,14	7,04	5,81	7,96	8,37	8,23	8,14
ΔMx [%]	3,15	4,39	5,83	7,10	7,79	3,33	4,58	5,98	7,18	7,75
My(I)	4,99	8,60	10,95	12,24	12,65	5,71	9,99	12,86	14,48	15,00
My(II)	5,26	9,12	11,67	13,09	13,54	6,05	10,63	13,75	15,52	16,09
ΔMy [%]	5,25	5,74	6,17	6,46	6,56	5,60	6,06	6,43	6,68	6,76
Node	1	2	3	4	5	6				
Bending moments [kNm/m]										
Mx(I)	0,00	-2,37	-4,10	-5,20	-5,79	-5,98				
Mx(II)	0,00	-2,48	-4,31	-5,49	-6,14	-6,34				
ΔMx [%]	0,00	4,50	4,92	5,33	5,63	5,76				
My(I)	0,00	-11,84	-20,50	-26,01	-28,97	-29,89				
My(II)	0,00	-12,40	-21,57	-27,47	-30,70	-31,72				
ΔMy [%]	0,00	4,50	4,92	5,33	5,63	5,76				

5. CONCLUSION

By establishing the balance conditions on the deformed plate, more real results of the force calculations and deformations are obtained. The geometric non-linear problem of bending the plate using a finite difference method is linearized. The solution of the linear equation system is determined by the iteration procedure. From the example shown, we conclude that the adopted initial imperfection of reinforced concrete slab significantly influences the results of the budget. The results of a second-order theory theory in relation to the first-order theory varying to ≈10% in the example shown.

REFERENCES

[1] Demirović, B.: Određivanje uticaja po teoriji drugog reda metodom konačnih razlika, *4. Međunarodna konferencija u Subotici*, **2016**., p.p. 341-347.;
[2] Hajdin, N.: *Teorija površinskih nosača*, Naučna knjiga, Beograd, **1989**., str. 63;
[3] Maglajlić, Z.: *Metoda relaksacije*, Građevinski fakultet u Sarajevu, Sarajevo, **2008**., str. 10-24;
[4] Timošenko, S., Vojnovski-Kriger, S.: *Teorija ploča i ljuski*, Građevinska knjiga, Beograd, **1962**., str. 299-300;
[5] Timoshenko, S., Gere, J.: *Theory of Elastic Stability*, McGRAW-HILL, New York, **1985**., p.p. 333-335.

ANALIZA PLOČA PO TEORIJI DRUGOG REDA NUMERIČKIM METODAMA

Rezime: *U radu je prikazana analiza savijanja ploča opterećenih okomito i u pravcu srednje ravni ploče. Postupak je urađen numerički, primjenom metode konačnih razlika. Proračun pomjeranja ploče u tačkama diskretizacije je proveden iterativnim postupcima uzimajući u obzir uticaje drugog reda. Analizom elementa ploče u deformisanom položaju izvedena je diferencijalna jednačina savijanja ploče, u kojoj se uvodi doprinos presječnih sila na momente savijanja i deformacije. Prikazanim metodama proračuna modelira se geometrijska nelinearnost ploča. Kroz numeričke primjere je prezentiran postupak proračuna i izvršena analiza rezultata.*

Ključne riječi: *geometrijska nelinearnost, teorija drugog reda, metoda konačnih razlika.*