INFLUENCE OF THICKNESS OF COLUMNS ON LOCAL BUCKLING-MODE INTERACTION BY FINITE STRIP METHOD AND DUALITY

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Summary: *A folded plate structure is defined as a prismatic structure which can be formed by folding a flat rectangular plate along lines parallel to its length. The thickness is small compared to its other dimensions and deformations are not large compared to thickness.*
These structures are particularly useful when the material is a composite laminate. The reason why it is usually more convenient to resort to numerical methods for these structures is that they are used in practical situations where there is a need to introduce irregularities like damage. It is found that a specialized finite element called the finite strip method (FSM) is very powerful, easier to set up, and gives accurate results [1].
In this paper duality of buckling and vibration phenomena is proved for the both elastic and viscoelastic (or damage) structure using rheological-dynamical analogy (RDA).
The governing dynamic RDA modulus has been derived in [2]. This paper presents an investigation of influence of thickness of composite thin-walled wide-flange columns on their response for buckling and vibration as well as local buckling-mode interaction. Numerical examples showing the theoretical considerations are presented.
A reference Open Source Software implementation for parametric modeling of local buckling and free vibration in prismatic structures is provided. It utilizes task-oriented local-level parallelisms in a form of multithreading/multiprocessing to overpower the increased computational complexity, caused by coupling a high number of series terms within a multi-dimensional parameter sweep (over thickness and length).

Keywords: *Duality, FSM, RDA, Dynamic RDA modulus, Composite thin-walled wide-flange column, multitreding, parallelisation*

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1. INTRODUCTION

When a polymer resin is reinforced with fibrous material, it is called a composite material or a Fiber Reinforced Plastic (FRP). Composite structural shapes are produced by pultrusion, with the geometry and material properties of the cross-section being fixed by the manufactured. A broad selection of such shapes is offered. They are used because of their high strength to weight ratio, resistance to environmental deterioration, and lack of interference with electromagnetic radiation. Since composite columns are thin-walled, buckling is a major consideration in design. Two types of column failure (buckling) are well known: local (web) and global (Euler) column buckling. Interaction between the local (web) and global (Euler) buckling modes occurs in intermediate length thin-walled columns with near coincident buckling loads [3].
Local buckling occurs in short columns that are long enough not to fail due to crushing; that is when the compression strength of the material is not reached. For pultruded wide-flange (WF) sections, Fig. 1, the column will compress axially until web develop wave like deformations along the length. The web deformations can be large, often greater than the thickness t_w of the web. The width of the web is b_w.Therefore the local buckling load can be used as a failure criteria for a short column.

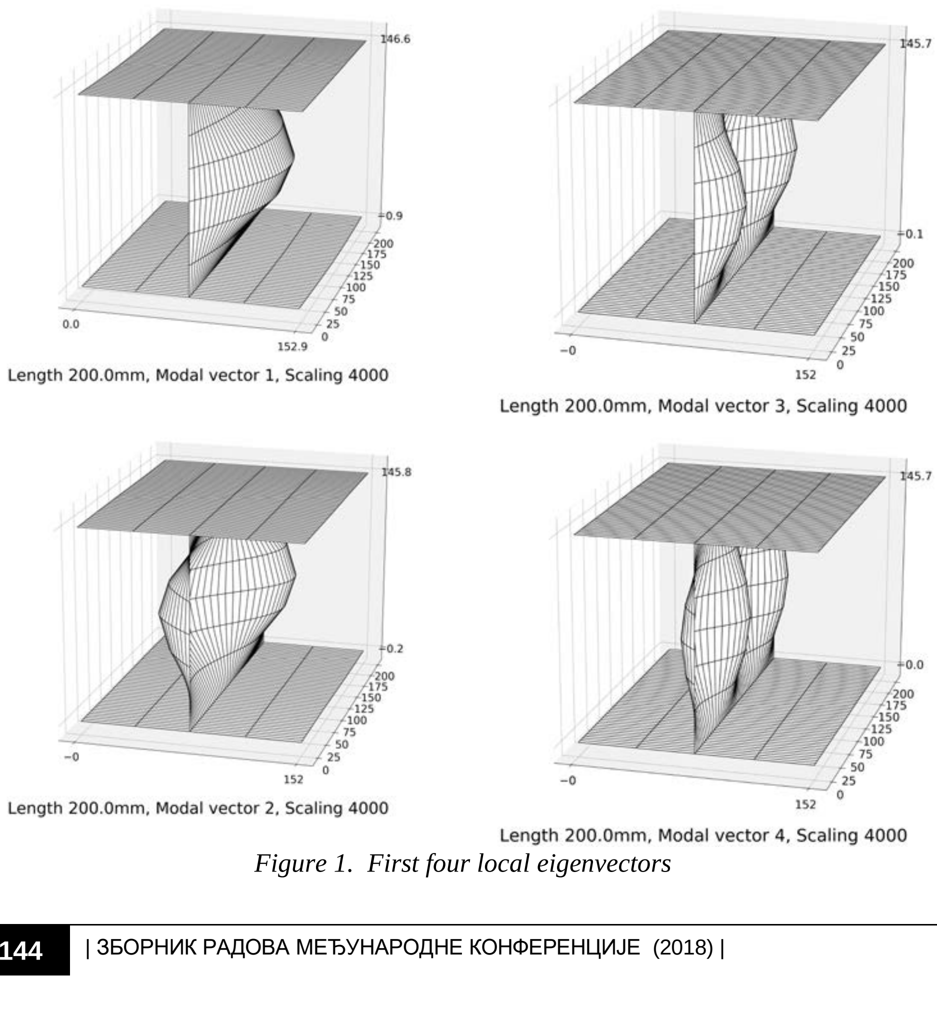

Figure 1. First four local eigenvectors

If the web is simply supported at its ends the well known critical stress for local mode is

$$\sigma_{cr} = \frac{\pi^2 E_H}{3\left(1-\mu^2\right)}\left(\frac{t_w}{b_w}\right)^2 \tag{1}$$

where E_H and μ are the modulus of elasticity and the Poisson's ratio of the material.

2. COINCIDENT BUCKLING STRESSES

In thin-walled columns and in stiffened plates, the interaction of global and local buckling modes may occurs under certain conditions of the design parameters. Let us take for example the simple case of an axially compressed wide-flange column represented in Fig. 1. The column is assumed to be simply supported at its ends. The Euler (global) buckling equation

$$\sigma_E = \frac{\pi^2 E_H}{\left(a/i\right)^2} \tag{2}$$

accurately predicts the critical buckling stress for slender columns in terms of the column length *a*, and the weak axis radius of gyration *i* of the cross-section.

The RDA states that modulus of elasticity in Eq. (1) must be replaced by a dynamic RDA modulus, treated as a viscoelastic one, because the web develop wave like deformations along the length, Fig. 1. Milašinović [2] has been highlighted the dependence of the Poisson's ratio on the viscoelastic creep coefficient

$$\varphi = \frac{2\mu}{1-2\mu} \tag{3}$$

where μ is the Poisson's ratio. The RDA modulus has already been derived as follows

$$E_R = \frac{1+\varphi+\delta^2}{\left(1+\varphi\right)^2+\delta^2}E_H\,, \quad \underset{lim\delta\to 0}{E_R} = E_R = \frac{E_H}{1+\varphi}, \quad \delta = \frac{\omega_\sigma}{\omega}. \tag{4}$$

where ω is the natural frequency of the mode and ω_σ is the frequency of excitation. The critical stress of local mode becomes

$$\sigma_{cr} = \frac{\pi^2 E_H}{3\left(1-\mu^2\right)\left(1+\varphi\right)}\left(\frac{t_w}{b_w}\right)^2. \tag{5}$$

For long composite columns, the Euler buckling is expected to occurs before any other instability failure. For short columns, local buckling occurs first, leading either to large deflections and finally to global buckling, or to material degradation due to large deflections.

When the length of the column is such that the predicted local and Euler loads are close, the experimental failure load may be lower than both predictions, depending on the imperfections, due to mode interaction between the local and global bucklings. The two buckling modes are coincident under the condition defined by Milašinović [3]

$$\frac{\pi^2 E_H}{(a/i)^2} = \frac{\pi^2 E_H}{3\left(1-\mu^2\right)(1+\varphi)}\left(\frac{t_w}{b_w}\right)^2 . \tag{6}$$

where the web width b_w is

$$b_w = b - \frac{3}{2} t_w . \tag{7}$$

Consequently, the creep coefficient is as follows

$$\varphi = \frac{(a/i)^2}{3\left(1-\mu^2\right)}\left(\frac{t_w}{b_w}\right)^2 - 1 . \tag{8}$$

3. SOFTWARE IMPLEMENTATION

The fsm_eigenvalue project [4] provides a reference Open Source Software implementation for parametric modeling of buckling and free vibration in prismatic structures, performed by solving the eigenvalue problem in the harmonic coupled finite strip method (HCFSM).

The fsm_eigenvalue project is composed of the following loosely-coupled software modules:

- ‘.load’, responsible for loading the parametric model data file
- ‘.compute’, responsible for solving the HCFSM eigenvalue problem and modeling of buckling and free vibration
- ‘.compute.integral_db’, responsible for downloading and working with the precomputed HCFSM integral tables [5]
- ‘.compute.matrices’, responsible for computing the local/global stiffness/stress/ mass matrices
- ‘.compute.parameter_sweep’, responsible for performing the multi-dimensional parameter sweep and distributing its workload onto multiple CPUs
- ‘.store’, responsible for storing the computed results
- ‘.main’, responsible for gluing the above modules together
- ‘.shell', responsible for providing the applications command-line interface

The fsm_eigenvalue project takes the semi-analytical finite strip model data file (geometry, materials, loading) as its input and then performs its computations as a parameter sweep over 4 separate dimensions: D1: Performs the buckling and free vibration analysis; D2: Iterates over all strip lengths, in the range specified by the input; D3: Iterates over all strip thicknesses, in the range specified by the input; D4: Iterates over all modes, in the range specified by the input. The total number of iterations is then D1 x D2 x D3 x D4, and they can be computed independently of each other, making this an “embarrassingly-parallel” problem. The ‘.compute.parameter_sweep’ module utilizes task-oriented local-level parallelisms in form of multithreading/multiprocessing to combat the increased computational complexity.

However, during development it was found that this 4-D iteration split is problematic because the resulting iterations were too fine-grained (due to their sub-second execution time) and much of the execution time was spent on synchronizing tasks and their results. Because of this we decided to make the iterations coarser by collapsing some of dimensions together with the goal of increasing the individual iteration time and decreasing the proportion of time spent on task synchronization.

The '.compute.parameter_sweep' module will also reorganize and group individual iterations into task "chunks", to further manage the inter-process communication overhead. It will try to determine the optimal number of chunks dynamically, through a heuristic based on the number of dimension-specific iterations within the parametric model data file.

During development it was found that even with all of the optimizations listed above the total number of iterations can still be very large – the fsm_eigenvalue project provided example data files have over 1,000,000 iterations each.
Therefore, the '.compute.parameter_sweep' and '.store' modules were reworked to use Python's generator/iterator protocols when computing iterations and collecting their results. In simplest terms, the '.compute.parameter_sweep' module computes the parametric model iterations and immediately "yields" their individual results through an iterator,while the '.store' module "listens" to this iterator. This effectively means that the fsm_eigenvalue project can write out its results just-in-time as they're computed, without waiting for the entire list of results to complete (or even actually forming the list for that matter). Thanks to the expressive power of the Python programming language this optimization has been implemented in less than 20 lines of code.

4. APPLICATIONS

The existence of buckling-mode interaction has been experimentally verified for intermediate length putruded wide-flange columns subjected to uniaxial compression [6] (Barbero at al., 2000). Characterization of the interaction mode has been done both conventional testing techniques and the shadow moire optical technique, allowing for non-contact, full field measurement of the buckling modes.
The material properties for the implementation of HCFSM and FSM are presented in [3]. In this section we analyzed the influence of thickness of columns on the interaction of more than one local buckling modes using the FSM and duality. H-section column is divided into 14 finite strips with 15 nodal lines. The main role has eigenvalues and corresponding local eigenvectors, Fig. 1. Though elastic buckling information for thin-walled members is not a direct predictor of capacity or collapse behaviour on its own, both the mode and the load are important proxies for the actual behaviour. In current design codes, such as AISI S100, New Zealand/Australia, and European Union, the design formulae are calibrated through the calculation of elastic critical buckling loads to predict the ultimate strength, thus the ability to calculate the associated elastic buckling loads is of great importance. Moreover, the buckling mode shapes are commonly employed into non-linear collapse modelling as initial geometric imperfections.

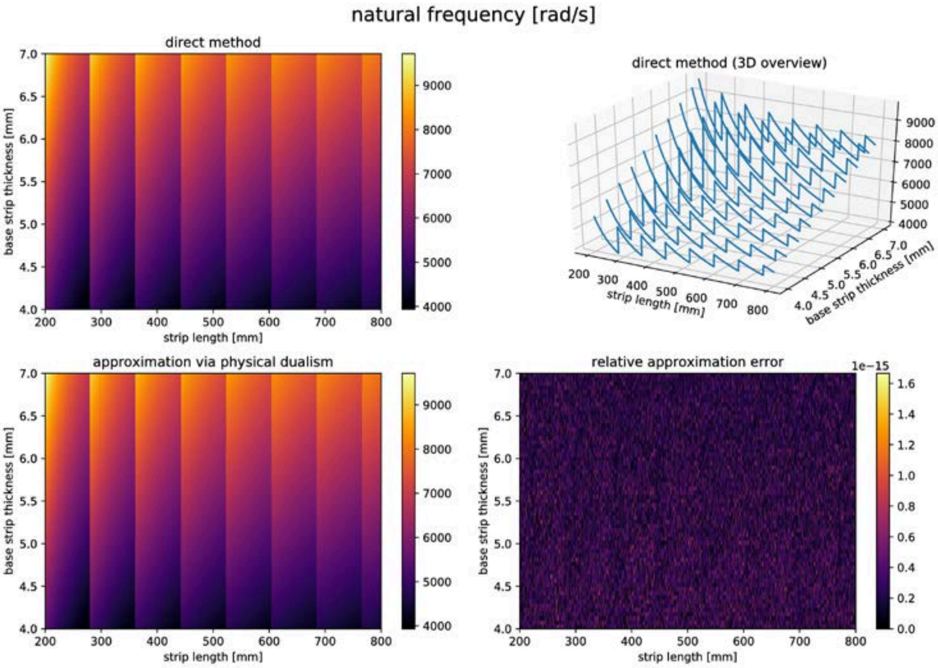

Figure 2. Elastic natural frequencies

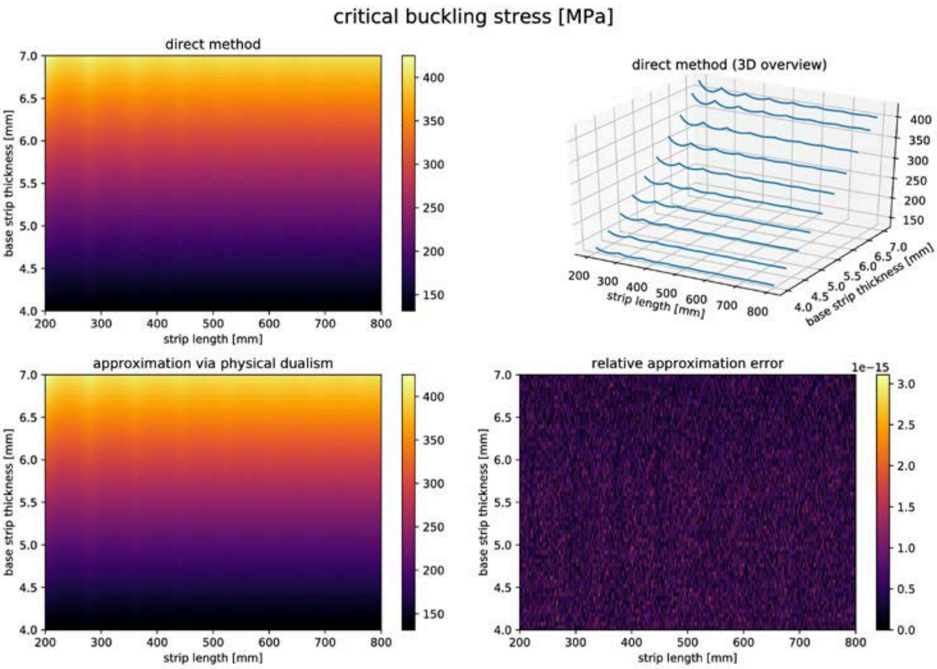

Figure 3. Elastic critical buckling stresses

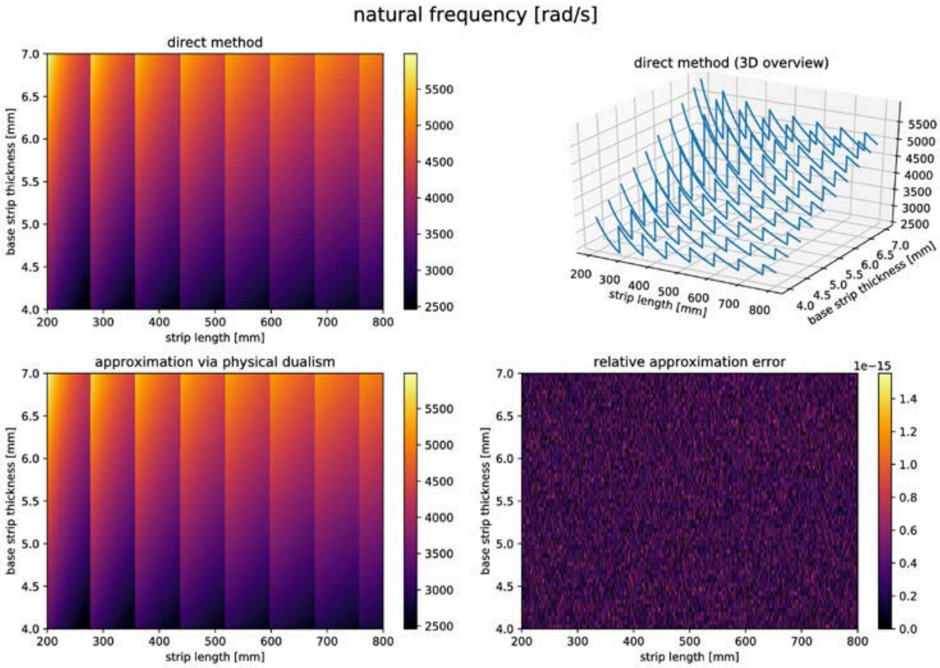

Figure 4. Viscoelastic natural frequencies

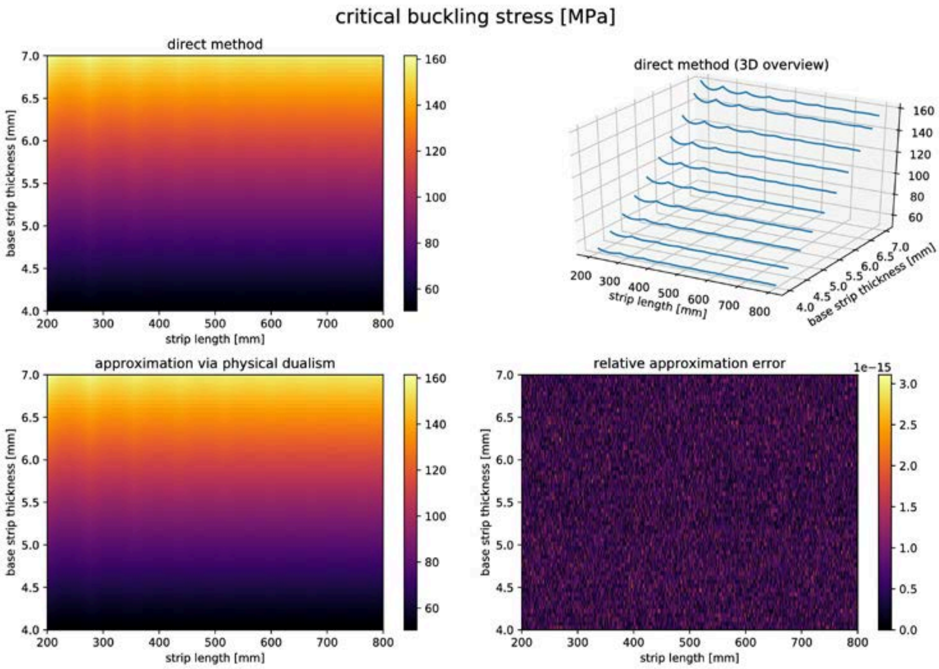

Figure 5. Viscoelastic critical buckling stresses

Fig. 2 show results of elastic natural frequencies for the column lenhts which has been successively increased from 200 to 800 mm, with a step of 0.5 mm. Thickness t has been varied from 4 mm to 7 mm, with a step of 0.5 mm. The frequency curves have jumps at the end of examined column length intervals, in contrast to the buckling curves, Fig. 3. It is clearly seen that the jumps of natural frequencies are always at the lengths of the columns on which the buckling curves has mode intersections. Such jumps within frequency curves are noticeable in both the elastic and viscoelastic solutions, Fig. 4. Also, there is a noticeable lag between the viscoelastic and the elastic natural frequencies.

Viscoelastic buckling stress, Fig. 5 lags behind the elastic buckling stress, Fig. 4 across all modes, which is a consequence of the viscoelastic behavior of materials, analogue to the described frequency behavior. The viscoelastic behavior is characterized by the delay tim T^D. As the length of the column is larger, the observed lag increases. Figs. 2, 3, 4 and 5 compare the values of viscoelastic natural frequencies as solved by the eigenvalue problem and the frequencies as computed by applying the critical stresses in equation of physical duality (and vice versa). Also, Figs. present the relative errors between the solutions as obtained by the eigenvalue problem and the solution as computed by physical duality.

The relative error of critical stresses is higher that the relative error of natural frequencies, because the buckling curve is curve of a higher order than the natural frequencies curve. The maximum relative error of only □ 1% confirms the physical duality approach and effectively shows that it is sufficient to solve th eigenvalue problem only for natural frequencies, while the critical stresses may be accurately computed through the equation of physical duality.

$$\omega_{im} = \frac{\sigma_{im}}{\rho}\left(\frac{m \cdot \pi}{a}\right)^2 . \tag{9}$$

where ω_{im} = natural frequency, i = number of degrees of freedom (DOF), m = number of series term, σ_{im} = critical stress, ρ = mass density, and a = length of the column. In this paper duality is proved for the both elastic and viscoelastic structure using the FSM and RDA.

5. CONCLUSIONS

Numerical examples were computed by application of extensive hybrid parallelization. Results from the numerical studies for all lengths of columns from 200 to 800 mm, show that columns have all elastic and inelastic characteristics in the local modes. However, the columns do not have all elastic and inelastic characteristics in the same mode. Because of that the local mode interactions are occurred at mode intersections for all thickness varied from 4 mm to 7 mm. The jumps of natural frequencies are always occured at the lengths of the columns on which the buckling curves has mode intersections.

Two reference Open Source Software implementations are provided: for approximating the natural frequency from stress via physical duality (and vice versa). In this paper duality is proved for the both elastic and first time for viscoelastic columns using the RDA.

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УТИЦАЈ ДЕБЉИНЕ СТУБОВА НА ЛОКАЛНУ МОДАЛНУ ИНТЕРАКЦИЈУ ПРИ ИЗВИЈАЊУ МЕТОДОМ КОНАЧНИХ ТРАКА И ДУАЛИТЕТОМ

***Резиме**: Полиедарска плочаста конструкција је дефинисане као призматична конструкција која може бити формирана савијањем равне правоугаоне плоче дуж линија паралелних дужини. Дебљина је мала у поређењу са њеним другим димензијама, а деформације нису велике у односу на дебљину. Ове конструкције су посебно корисне када је материјал композитни ламинат. Разлог зашто је најчешће лакше разматрати ове конструкције нумеричким методама је тај што се оне у практичним ситуацијама користе када је неопходно унети неправилности, као што су оштећења. Утврђено је да специјализовани коначни елемент, назван метод коначних трака (МКТ), је веома моћан алат, лакши за подешавање и да даје прецизне резултате [1]. У овом раду далитет појава извијања и вибрација је доказан за обе еластичну и вискоеластичну (или оштећену) конструкцију помоћу реолошко-динамичке аналогије (РДА). Владајући* *динамички РДА модул је изведен у [2]. Овај рад представља истраживање о утицају дебљине композита танкозидних стубова са широким фланшама на њихов одговор при извијању и вибрацији као и локалној модалној интеракцији при извијању. Развијен је софтвер отвореног кода за параметарско моделирање локалног извијања и слободне вибрације призматичних конструкција. Он користи паралелизам на локалном нивоу, који је оријентисан према задатку и заснован на употреби више нити/процеса, ради савладавања повећане прорачунске комплексности, узроковане више-димензионалном параметарском анализом (преко дебљине и дужине) великог броја везаних низова података.*

Кључне речи: *Дуалитет, МКТ, РДА, динамичи РДА модул, композитни танкозидни стуб са широким фланшама, мултитрединг, паралелизација*