NUMERICAL MODELLING OF A STEEL STRUT SUBJECTED TO AXIAL COMPRESSION

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Summary: *Steel constructions include axially compressed elements that require analysis of stability. Existing analytical solutions to the stability problem are mostly based on crude assumptions and do not adequately account for the geometrical imperfections and semi-rigid connections of elements. In this paper, a railway bridge truss strut subject to compression is analysed by the finite difference method and the second order theory, and values for stresses, displacements and element effective buckling length are obtained.*

Keywords: *Buckling, FDA, semi-rigid connections, geometrical imperfection*

1. INTRODUCTION

Buckling phenomenon and numerical values for ideal members have been firstly established by Swiss mathematician *Euler* in 1744. Considering assumption for elastic behaviour of material and ideal boundary conditions (rigid or hinge constrains) known relations for critical load can be derived for ideal member. Also, constrains that applies for bifurcation stability problem are adopted [1].

As the most of the real materials, steel is elastic-plastic material, and it has elastic properties until it reaches yield strength. When stresses exceed yield strength plastic behaviour occurs. In plastic domain assumptions about linear elastic stress-strain relation does not apply. The first equations that describe behaviour in this domain were established by *Tetmeyer*, and significant contribution to the theory of buckling was provided by *Bauschinger*, *Engesser*, *Karman* and *Shanley* [1], [2].

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In this paper buckling case of a steel truss bridge strut was analysed according to the example given in literature [3]. The bridge spans L=50.0 m, it is horizontal and straight. The main girders are trapezoidal trusses with webs that forms triangular panels (Fig. 1). The trusses are simply supported beams. Height of the trusses is 7500 mm, and the bridge width is 5000 mm. Parent material for the structure is structural steel S235 [3].

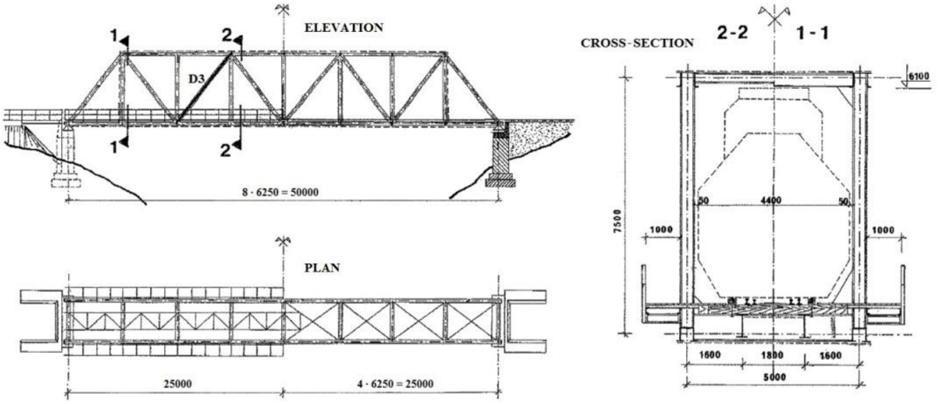

Figure 1. General view of the bridge [3]

The strut D_3 is analysed because degrees of fixity are approximately the same at both ends. Linear static analysis showed that design value of force in strut is $N_{D,3}$=-1048.9 kN. Design is performed according to SRPS U.E7.081/1986, and for adopted cross section of the strut utilization rate regarding allowed stress is 72.8 % [3].

2. MATHEMATICAL MODEL OF THE STRUT

Imperfect member with length of 2*l* and rigidity *EI* is considered. The member deviates from ideal straight line for y_{max}^{imp} in the middle (geometrical imperfection) and it has semi-rigid connections with supports (Fig. 2). In deformation estimation only bending moment is taken into account, and normal and transverse forces were not considered.

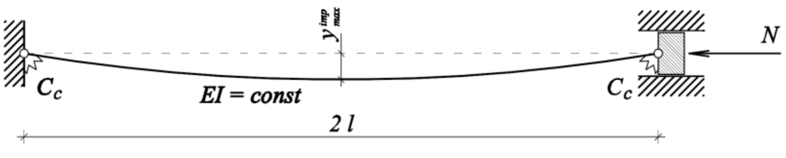

Figure 2. Model of strut (without deformation)

Problem can be simplified by using symmetry simplification. Coordinate system can be placed in the middle of the member, so that deflection of the middle of the member, is actually deflection of the end of the member that occurs because of external loading. In this way the model beam (analysed member) has the following boundary conditions: at one end the member is fixed, and on the other end displacements are free, but the bending moment occurs (Fig. 3).

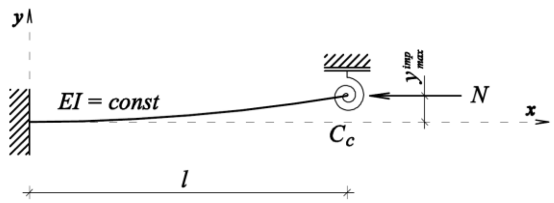

Figure 3. Simplified model (without deformation)

For numerical analysis it is necessary to divide the member at n equal segments. The number of the segments depends on the desired calculation accuracy and available time for calculation. It is estimated that preferred number of segments should be $n = 300 \div 1000$, and for this paper the adopted number of segments is $n = 500$.

3. GEOMETRICAL IMPERFECTIONS AND RESIDUAL STRESSES

Because of the steel processing (rolling, welding) random structural and geometric imperfections occurs. Significant problem can be residual stresses caused by mentioned processing. For simplified calculations, standards impose allowed deviations. Allowed geometrical deviation from the straight line can be determined as:

$$\delta_{0,max}^{geom} = \frac{l}{1\,000} \tag{1}$$

where l represents the member length.

Residual stresses caused by rolling and welding requires much more complex analysis than needed for practical application. In 1978. *Maquoi* and *Rondal* provided method for estimating total imperfection of the member [4]. Regarding the cross section types, the members can be divided into five groups, and each of these groups has defined equivalent imperfection degree (α). Total starting imperfection (δ_0) can be calculated using these relations:

$$\delta_0 = y_{max}^{imp} = \alpha\left(\overline{\lambda} - 0.2\right)\frac{W}{A} \tag{2}$$

$$\overline{\lambda} = \sqrt{\frac{A\,f_y}{N_{cr}}} \tag{3}$$

$$N_{cr} = \frac{\pi^2\,E\,I}{{l_0}^2} \tag{4}$$

where: W – section modulus;
A – cross section area;
f_y – steel yield strength;
N_{cr} – *Euler* critical load.

By introducing corresponding geometric values of the member in equations above, the total imperfection can be calculated $\left(y_{max}^{imp}\right)$. Assuming that the centroid axis of the member follows the second order parabolic line, and that function argument is a whole number as multiplier of the segment, those equations can be derived:

$$x = i\,\Delta x = i\,\frac{l}{n} \tag{5}$$

$$y^{imp}(x) = p\,x^2 \tag{6}$$

Using boundary conditions $y^{imp}(0) = 0$ and $y^{imp}(l) = y_{max}^{imp}$, the unknown parameter p and deviation value for node i can be derived:

$$y^{imp}(x) = y_{max}^{imp}\,\frac{x^2}{l^2} = y_{max}^{imp}\,\frac{(i\,\Delta x)^2}{(n\,\Delta x)^2} = y_{max}^{imp}\,\frac{i^2}{n^2}. \tag{7}$$

4. BASIC EQUATION DERIVATION

A cantilever with a spring on its free end, loaded with concentrated load, is considered. The equation for bending moment at node i can be derived. Rotational spring influence can be taken in consideration by imposing a bending moment $M = C_c\,\varphi$, where coefficient C_c is equal to the bending moment that would rotate the spring for one rotation unit angle (Fig. 4).

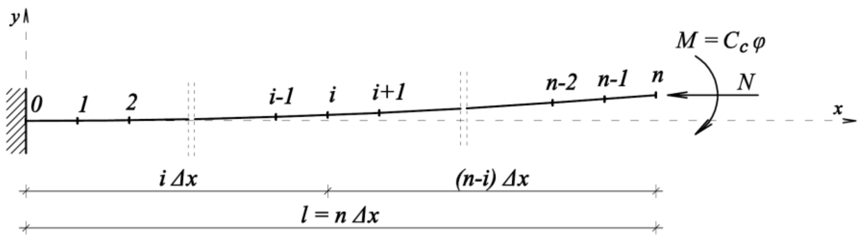

Figure 4. Discretized model with loads

$$M_i = N\,(y_n^{tot} - y_i^{tot}) - C_c\,\varphi = N\,\left(y_n + y_n^{imp} - y_i - y_i^{imp}\right) - C_c\,\frac{y_n - y_{n-1}}{\Delta x} \tag{8}$$

In this equation: y_n^{tot}, y_i^{tot} – distance of nodes n and i from the direction x after deformation;
y_n, y_{n-1}, y_i – displacements of n, $n-1$ and i node by loading.

Deformation equation of the member can be determined by calculating the displacement of node $i+1$, if deviation, tangent inclination and value of bending moment is known for node i (Fig. 5).

$$y_{i+1} = y_i + \Delta y_i' + \Delta y_i(M) \tag{9}$$

Displacement increment is equal to the sum of the previous increment and additional increment (caused by bending moment). Value $\Delta y_i'$ can be determined as a difference of two previous known displacement:

$$\Delta y_i' = y_i - y_{i-1}. \tag{10}$$

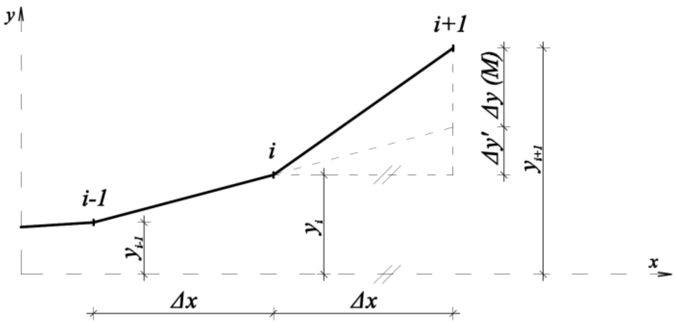

Figure 5. Calculation of the displacement increment for node i+1

In this case additional increment is considered taking only the bending moment in calculation. For more accurate analysis it is necessary to take into account the shear force and the temperature difference influence. The displacement caused by bending moment represents majority of the total transverse displacement for the members mainly subjected to bending, so it is reasonable to disregard other influences.

If cantilever with length of Δx, subjected to bending moment M_i at free end is considered, using the principle of virtual forces and the principle of superposition, the angle of rotation of free end can be derived, and also the angle increment of nodes i and $i+1$:

$$\Delta\varphi_i = \frac{M_i\,\Delta x}{EI}. \tag{11}$$

Using assumption of small angles $(\sin\varphi = \operatorname{tg}\varphi = \varphi)$, additional increment $\Delta y_i(M)$ can be derived by multiplying the previous equation with Δx:

$$\Delta y_i(M) = \Delta\varphi_i\,\Delta x = \frac{M_i \Delta x^2}{EI}. \tag{12}$$

By replacing equations (10) and (12) into the equation (9) relation between displacement of three successive nodes is:

$$y_{i+1} - 2\,y_i + y_{i-1} - \frac{M_i\,\Delta x^2}{EI} = 0. \tag{13}$$

When relations (8) and (7) are introduced into the previous equation and expressed in appropriate form for matrix calculus, it becomes:

$$\begin{aligned} &y_{i-1} + y_i\left(-2 + \frac{N\,\Delta x^2}{EI}\right) + y_{i+1} + y_{n-1}\left(-\frac{C_c\,\Delta x}{EI}\right) + \\ &+y_n\left(-\frac{N\,\Delta x^2}{EI} + \frac{C_c\,\Delta x}{EI}\right) + y_n^{imp}\frac{N\,\Delta x^2}{EI}\left(-1 + \frac{i^2}{n^2}\right) = 0 \end{aligned} \tag{14}$$

If the term next to y_i is marked as k_i, the term next to y_{n-1} as $\overline{k}_{n-1}$, term next to y_n as $\overline{k}_n$, and free term as g_{i+1}, previous equation can be written as:

$$y_{i-1} + k_i\,y_i + y_{i+1} + \overline{k}_{n-1}\,y_{n-1} + \overline{k}_n\,y_n + g_{i+1} = 0. \tag{15}$$

The equation can be written for $i = 1, n-1$, but for $i = 0$ and $i = n$ this cannot be done because previous and next node does not exist.

5. BOUNDARY CONDITION AND MATRIX EQUATION

Conditional equation for $i = 0$ can be written by considering the first cantilever $(0-1)$. Displacement y_1 can be calculated as a deflection of the free end, if cantilever is subjected to bending moment at that point only. By using the principle of virtual forces for deflection determination, and condition of indivertible node $i = 0$ ($y_0 = 0$), it can be calculated that:

$$y_1 = \frac{M_0 \Delta x^2}{2\,EI} \tag{16}$$

and, if equations (8) and (7) are introduced into equation (16), the next equation can be derived:

$$y_1 + y_{n-1}\left(-\frac{C_c\,\Delta x}{2\,EI}\right) + y_n\left(-\frac{N\,\Delta x^2}{2\,EI} + \frac{C_c\,\Delta x}{2\,EI}\right) + y_n^{imp}\frac{N\,\Delta x^2}{2\,EI}(-1) = 0. \tag{17}$$

As for basic equation, the terms next to the unknown and free term are necessary to mark, but now instead of $\overline{k}_{n-1}$, $\overline{k}_n$ and g_{i+1} their half values emerge. Hence, relation (17) becomes:

$$y_1 + y_{n-1}\frac{\overline{k}_{n-1}}{2} + y_n\frac{\overline{k}_n}{2} + \frac{g_1}{2} = 0. \tag{18}$$

System of equations (15) and (18) can be written, for the purpose of a more convenient implementation, in the matrix form, taking into consideration that in the last two rows some elements should be added up because of interference in the matrix.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & \frac{\bar{k}_{n-1}}{2} & \frac{\bar{k}_n}{2} \\ 1 & k_i & 1 & 0 & \cdots & 0 & \bar{k}_{n-1} & \bar{k}_n \\ 0 & 1 & k_i & 1 & \cdots & 0 & \bar{k}_{n-1} & \bar{k}_n \\ 0 & 0 & 1 & k_i & \cdots & 0 & \bar{k}_{n-1} & \bar{k}_n \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & k_i & 1+\bar{k}_{n-1} & \bar{k}_n \\ 0 & 0 & 0 & 0 & \cdots & 1 & k_i+\bar{k}_{n-1} & 1+\bar{k}_n \\ / & / & / & / & \cdots & / & / & / \end{bmatrix} \begin{Bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{n-2} \\ y_{n-1} \\ y_n \end{Bmatrix} + \begin{Bmatrix} \frac{g_1}{2} \\ g_2 \\ g_3 \\ g_4 \\ \vdots \\ g_{n-1} \\ g_n \\ g_{n+1} \end{Bmatrix} = 0 \tag{19}$$

The matrix of coefficients in matrix equation (19) misses last row. Boundary condition for node $i = n$ is not possible to write because the deflection y_n is not limited. But, the equation can be simplified using known deflection $y_0 = 0$. By taking out the first column and last row of matrix of coefficients, and taking out the term y_0 from vector of deflection and term g_{n+1} from vector of free terms, system of equation in order of n is obtained:

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & \frac{\bar{k}_{n-1}}{2} & \frac{\bar{k}_n}{2} \\ k_i & 1 & 0 & \cdots & 0 & \bar{k}_{n-1} & \bar{k}_n \\ 1 & k_i & 1 & \cdots & 0 & \bar{k}_{n-1} & \bar{k}_n \\ 0 & 1 & k_i & \cdots & 0 & \bar{k}_{n-1} & \bar{k}_n \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & k_i & 1+\bar{k}_{n-1} & \bar{k}_n \\ 0 & 0 & 0 & \cdots & 1 & k_i+\bar{k}_{n-1} & 1+\bar{k}_n \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{n-2} \\ y_{n-1} \\ y_n \end{Bmatrix} + \begin{Bmatrix} \frac{g_1}{2} \\ g_2 \\ g_3 \\ g_4 \\ \vdots \\ g_{n-1} \\ g_n \end{Bmatrix} = 0 \tag{20}$$

If the matrix of coefficients is marked with $[\mathbf{K}]$, the vector of unknown deflections with $\{\mathbf{y}\}$, and vector of free terms with $\{\mathbf{g}\}$, system (20) can be presented in simple form:

$$[\mathbf{K}]\{\mathbf{y}\} + \{\mathbf{g}\} = 0 \tag{21}$$

Equation (21) has a unique solution and its solving results with n unknown deflections:

$$\{\mathbf{y}\} = -[\mathbf{K}]^{-1}\{\mathbf{g}\} \tag{22}$$

By replacing the calculated deflections into the relation (8), bending moments at nodes and stress values at specific points can be obtained. If distribution of bending moments along the member is known, buckling length can be obtained by finding the zero point.

6. STRESS DETERMINATION IN BRIDGE STRUT

In order to constitute and solve the matrix equation, a program *Buck_memb* has been written using the software $Mathematica$ 7.0. The design for the strut in analysed example [3] is provided and welded box-shaped cross-section is adopted (Fig. 6).

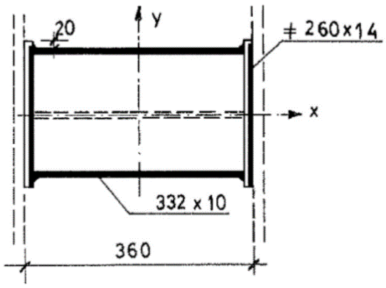

Figure 6. Cross-section of the strut D3

Properties: $2\ l = 2 * 4\ 881.5\ mm = 9\ 763\ mm$ – length of the strut;
$A = 139.2\ cm^2$;
$I_x = 11\ 422\ cm^4$;
$W_x = 878.6\ cm^3$;
$E = 21\ 000\ \ kN/cm^2$;
$f_y = 24\ \ kN/cm^2$.

Using relations (2), (3) and (4), total geometrical imperfection for the analysed member is obtained as $\delta_0 = 2.96\ cm$. According to the type of the cross-section of the member the value of the buckling curve coefficient [5] is $\alpha = 0.489$.

Rotational rigidity of the connection at the strut joint is determined by approximate method, using displacement capacity of a fitted bolt under shear load. The connection is designed with 16 $M24$ bolts with grade 10.9, with double gusset plate $18\,mm$ thick (Fig. 7). The model was analysed using *Tower 7.0 Demo* software and rigidity of the connection is estimated as $C_c = 540\ 000\ \ kNcm/rad$.

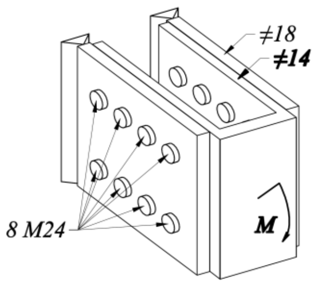

Figure 7. Model of the joint connection (drawing)

Using the programme *Buck_memb* for different values of compression force, lateral displacements, bending moments and stresses are obtained. Result are shown in Table 1 and graphically (Fig. 8). Verification of the calculation is done using software *Tower 7.0 Demo.*

Table 1. Displacement, bending moment and normal stress for different values of compression force

	N [kN]	**y_{max} [mm]**	**M_{max} [kNm]**	**σ_{max} [kN/cm^2]**
Buck_memb	0.0	0.00	0.00	0.00
	500.0	4.04	11.01	4.84
	1 000.0	9.32	25.58	10.10
	1 500.0	16.52	45.69	15.98
	2 070.7	28.74	80.25	24.01
Tower 7.0	2 070.7	28.70	80.17	24.00

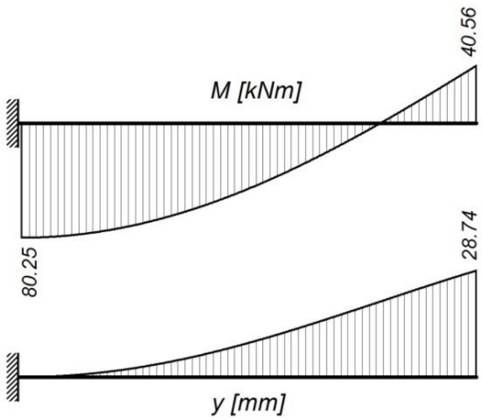

Figure 8. Distribution of the bending moment and displacements along the strut for applied force N=2070.7 kN

7. CONCLUSION

The highest value of plasticisation force in the strut is $N_{max} = 2\ 070.7\ kN$. Considering that governing load is combination of basic loads ($\nu^I = 1.50$), allowed force, and utilization rate of the strut D_3 is:

$$N_{dop} = \frac{N_{max}}{\nu^I} = \frac{2\ 070.7}{1.50} = 1\ 380.5\ kN; \quad \frac{N_{D_3}}{N_{dop}} = \frac{1\ 048.9}{1\ 380.5} = 76.0\ \% \qquad (23)$$

Distance from the null point of bending moment to the origin of coordinate system is $x_0 = 384.6\ cm$, so the buckling length coefficient has value (assumed value in example [3] is $\beta = 0.80$):

$$\beta = \frac{x_0}{l} = \frac{384.6}{488.15} = 0.79 \quad (24)$$

Application of numerical calculation methods is important in the cases of very complex problems, when analytical expression do not provide framework for influences of all relevant parameters [5]. When it comes to the simple girders the problem can be solved using second order theory equations, where material is assumed to be homogeneous, isotropic and elastic. However, if the plastic behaviour of material is included in the analysis, the problems is complicated to some extent, which calls for the application of numerical methods for the purpose of taking into account of all the relevant influences.

The proposed numerical procedure of determination of the stress state for axial loaded members allows implementation of the plastic and viscous behaviour of the member, whereby the problem is rendered somewhat more complex than in the case of the material having elastic properties, which was treated in this paper.

REFERENCES

[1] Buđevac, D., Marković, Z., Bogavac, D., Tošić, D.: *Metalne konstrukcije – Osnove proračuna i konstruisanja*, Građevinska knjiga, Beograd, **1999**.
[2] Gojković, M.: *Drvene konstrukcije*, Naučna knjiga, Beograd, **1989**.
[3] Buđevac, D., Stipanić, B.: *Čelični mostovi*, Građevinska knjiga, Beograd, **1989**.
[4] Maquoi, R., Rondal, J.: *Analytische formulierung der neuen europäischen knick spannungskurven*, Stahlbau 1, Germany, **1978**.
[5] Zarić, B., Buđevac, D., Stipanić, B.: *Čelične konstrukcije u građevinarstvu*, Građevinska knjiga, **2004**.
[6] Milić, M.: Primena metode konačnih razlika pri proračunu napona oko kružnog tunela, *6. internacionalni naučno – stručni skup Građevinarstvo – nauka i praksa*, Žabljak, Crna Gora, **2016**., p.p. 1439-1446.

НУМЕРИЧКО МОДЕЛИРАЊЕ ПРИТИСНУТЕ ДИЈАГОНАЛЕ ЧЕЛИЧНЕ РЕШЕТКЕ

Резиме: *Челичне конструкције често садрже аксијално притиснуте елементе који захтевају анализу стабилности. Постојећа аналитичка решења проблема стабилности углавном су засновања на грубим претпоставкама и не узимају у обзир геометријске несавршености елемената и полукруте везе између њих на адекватан начин. У овом раду је анализирана притиснута дијагонала железничког решеткастог моста применом методе коначних разлика и теорије другог реда, и добијене су вредности напона, померања и дужине извијања.*

Кључне речи: *Извијање, МКР, полукруте везе, геометријска имперфекција*