INTRINSIC PERMEABILITY BY RHEOLOGICAL DYNAMIC

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ABSTRACT:

Porosity and permeability are related properties of solids that are investigated by different continuum models. Porosity has already been explained by the author as the variation of the strain energy density of a dry solid using the rheological-dynamical analogy (RDA). Permeability is traditionally, theoretically and experimentally, investigated by Darcy's law and as such depends on the properties of the fluid, water or gas flowing through the solid. This means that we do not have a defined permeability of solid matter, which is independent of the properties of water or gas. In this paper, intrinsic permeability is defined based on the rheological dynamics theory (RDT) in the state of critical damping. Furthermore, RDT defines the liquid flow criterion by the porosity threshold of the solid. Permeability defined in this way is confirmed by experimental data from the literature on the example of engineered permeable concrete.

KEYWORDS:

Percolation, Permeability, Darcy's law, solid, critical damping, liquid, flow criterion.

1 INTRODUCTION

In physics, chemistry, and materials science, percolation refers to the movement and filtration of liquids through porous materials were permeability is key factor. Starting from the experimental point of view in permeability research [1], Darcy's law [2] from 1856 cannot be bypassed. The reason is that devices for measuring volumetric flow through a geometrically defined porous solid can easily be made. Although this point of view is physically understandable, Darcy's law remains empirical because the permeability coefficient of water or gas depends on the characteristics of the fluid flowing through the solid. Permeability is an important phenomenon, which often manifests differently in materials. For example, some rocks may have low porosity, but in the case of jointed fractures, their permeability is high. Conversely, some rocks may have high porosity, but in the case of unconnected pores, their permeability is low.

Although time is included in the analysis by Darcy's law by measuring fluid flow, the author of this paper is not aware that permeability research was understood as a rheological problem of solid. In fact, permeability should be investigated and formulated as the ability of a solid to allow a liquid to pass through free pores if the porous medium is understood as a damaged continuum. Because of that a porous medium, using damage mechanics [3], [4] must be considered when defining permeability. Apart from this, perhaps it is more important to examine the characteristics of the solid in terms of its strength, modulus of elasticity [5] and of course porosity [6].

Unfortunately, in the application of Darcy's law, the mechanical properties of the solid do not appear in the formulation, so the published works on this problem do not provide all the necessary mechanical properties of the porous medium. On the other hand, RDT properties in a state of critical damping [7] includes a large number of mechanical properties of solid as well as the fluid phase of this solid at the time of its natural formation or at the time of its preparation as an artificial material [8]. Taking all of the above into account, RDT emerges as an authoritative method for permeability research. This work is dedicated to that and can be considered a new approach, especially since all model parameters can be calculated from measured P and S wave velocities [9].

2 INTRINSIC PERMEABILITY BY RHEOLOGICAL DYNAMICS

The one-dimensional movement of fluid through the porous medium is laminar, and can be represented by Darcy's law, Figure 1:

$$Q = -A_0 \frac{k}{\eta} \frac{d\sigma}{dz} \tag{1}$$

where: Q [m^3/s] is the volumetric flow or flux (the negative sign in the Darcy equation is used so that a positive value of Q will indicate flow in the positive z direction), $\mathbf{A_0}$ [m^2] is the area of the specimen perpendicular to the flow direction, k [m^2] is the permeability, η

[Pa·s] is the viscosity of the permeating fluid and $d\sigma/dz$ [Pa/m] is the pressure gradient. For incompressible fluids the permeability k as given in Eq. (1) corresponds to the intrinsic permeability.

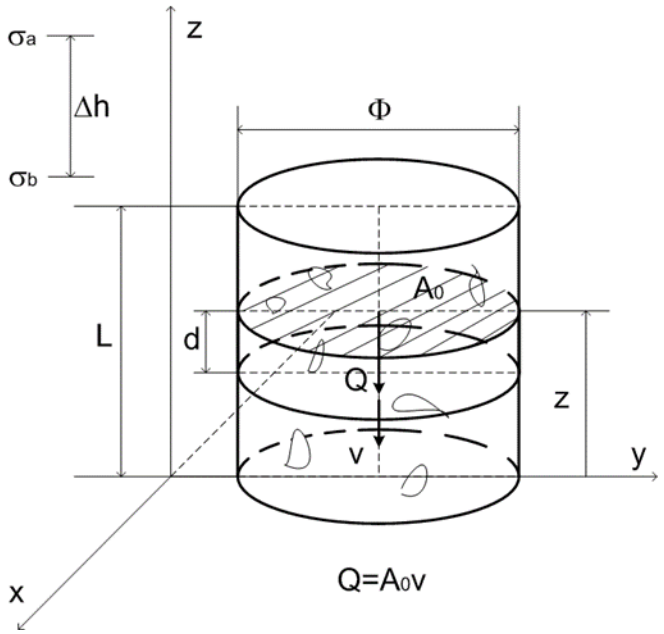

Figure 1: One-dimensional movement of fluid through the porous medium

Equation (1) can be integrated:

$$\frac{Q}{A_0}\frac{\eta}{k}\int_0^L dz = -\int_{\sigma_a}^{\sigma_b} d\sigma \Rightarrow \frac{Q}{A_0}\frac{\eta}{k}L = -(\sigma_b - \sigma_a) \tag{2}$$

So, the permeability is:

$$k = \frac{Q}{A_0}\frac{\eta}{\sigma_a - \sigma_b}L \tag{3}$$

The velocity of the material particles, that is, the solid and fluid phases of the elastic medium, is defined as follows [10]:

$$v = \frac{\sigma_a - \sigma_b}{\sqrt{E_H \rho}} \tag{4}$$

Where E_H is the modulus of elasticity while ρ is the density of dry solid. Substituting into Eq. (3) gives the intrinsic permeability:

$$k = \frac{\eta}{\sqrt{E_H \rho}}L \tag{5}$$

According to RDT, the effect of porosity is defined as explained in [6]:

$$k = \frac{\eta}{\sqrt{E_{H,0}\left(1-\frac{p}{p_{\max}}\right)\rho_0(1-p)}}L \tag{6}$$

where: p is the porosity, p_{max} is the maximum porosity, while $E_{H,0}$ and ρ_0 are the modulus of elasticity and density with zero porosity.

According to RDT the mechanical properties of dry solids are determined by the velocities of P and S waves in the state of critical damping [10]. By including the critical plastic shear viscosity $\eta_{N,cr}$ in Eq. (6), we obtain the critical intrinsic permeability:

$$k_{cr} = \frac{\eta_{N,cr}}{\sqrt{E_{H,0}\left(1-\frac{p}{p_{\max}}\right)\rho_0(1-p)}}L \tag{7}$$

Consequently, intrinsic permeability depends only on the characteristics of dry solid. Moreover, as used above, the strain rate of solid controls the volumetric flow, $Q = A_0 v$ as follows:

$$\frac{d\varepsilon}{dt} = \frac{\sigma_a - \sigma_b}{E_H T^D} = \frac{\sigma_a - \sigma_b}{E_H L}\sqrt{\frac{E_H}{\rho}} \Rightarrow v = \frac{d\varepsilon}{dt}L = \frac{\sigma_a - \sigma_b}{\sqrt{E_H \rho}} \tag{8}$$

3 WATER PERMEABILITY BY RHEOLOGICAL DYNAMIC

In civil engineering problems attention has often to be given to the flow of water through porous soil. So, the permeability describes the ability of a porous soil to resist water ingress. Darcy's law is an empirical relationship, which is mathematically equivalent to Ohm's law or Fick's first law. Experiments show that even in scalar form, when second rank tensor k_{ij} becomes simply k, both fluid and porous medium characteristics are involved. So, taking into account Eq. (7) the water permeability is defined as follows:

$$k_w = \frac{\eta_w}{\sqrt{E_{H,0}\left(1-\frac{p}{p_{\max}}\right)\rho_0(1-p)}}L \tag{9}$$

where η_w is the water dynamic viscosity. It is evident that for any given soil k_w will depend upon the viscosity η_w and hence on the temperature of the water, but for practical purposes the variation in k_w with the range of temperature normally encountered in the ground is of little consequence. Since the temperature of ground water is usually about $10°C$ it is convenient to report the results of laboratory determinations of the water permeability coefficient K_w corrected to this temperature:

$$K_w = \frac{Q}{A_0 \Delta h} L \tag{10}$$

where Δh is the drop in hydraulic head, over a length L, measured in the direction of flow, Figure 1. Applying a constant pressure at the end faces of the sample shown in Figure 1 we obtain:

$$\sigma_a - \sigma_b = \Delta h \rho_w g \tag{11}$$

Substituting into Eq. (3) follows the water permeability:

$$k_w = \frac{Q}{A_0} \frac{\eta_w}{\Delta h \rho_w g} L = K_w \frac{\eta_w}{\rho_w g} \tag{12}$$

For water: ρ_w= 1000 kg/m^3, η_w=0.001 Pa·s. Therefore,

$$k_w = \frac{0.001}{1000 \cdot 9.81} K_w = 1.02 \cdot 10^{-7} K_w \tag{13}$$

4 EXPERIMENTAL VALIDATION

Permeable concrete is a multi-phase structure with voids in it, Figure 1. It is designed so that free water can easily pass through the voids of the solid medium, thus achieving the effect of permeability. Due to this condition, water flows through the interstices of the permeable solid structure and the effect of water leakage occurs.

4.1 PERMEABLE CONCRETE EXPERIMENTS

The strength of permeable concrete mainly comes from the point of contact between the aggregates. Because of that, prior to the permeability proof experiments [1], the aggregate was washed and the silt content was essentially zero. In [1], the particle size of 4,75 mm ~ 9,5 mm was used. Measured particle density was 2660 kg/m^3 while dry bulk density ρ was 1499 kg/m^3. So, the designed porosity was

$$p_{des} = 1 - \frac{\rho}{\rho_{ap}} = 1 - \frac{1499}{2660} = 0.436\,. \tag{14}$$

Also the cement, which is a component of permeable concrete, has a degree of strength, activity, variety and dosage. Because of that it is important factor in the permeability and strength. In [5], it was shown that the interface between the aggregates and the cement matrix and the strength of the cement layer between the aggregates are the key factors that influence the modulus of elasticity and the strength of permeable concrete. The smaller the aggregate particle, the smaller the pore, and the more cementation points between aggregates. In [5], cylindrical samples of Φ =50 mm and L =100 mm made of the aggregate described in [1] were used to investigate the effective porosity. A water-cement ratio of 0,3 was used without fine aggregates and concrete mixture were designed to achive the porosity of p_{ef}=0,18. When sample is formed, it is necessary to form a compact solid at the end faces of the sample, which has a great influence on the modulus of elasticity and uniaxial compressive strength. The modulus of elasticity of permeable sample was 7320 MPa while uniaxial compression strength was f_{cS}=19,5 MPa [5]. The inclined section shear failure of sample was occurred [5].

4.2 EFFECTS OF POROSITY BY RHEOLOGICAL DYNAMICS

In addition to the density ρ of the dry sample for a known geometry, data on the velocities of P and S waves are also required. The following velocities were calculated using RDT: v_L=2484 m/s and v_T=1384 m/s.

Therefore:

$$E_D = v_L^2 \rho = 9250 \text{ MPa}, \tag{15}$$

$$G_H = v_T^2 \rho = 2871 \text{ MPa}, \tag{16}$$

$$E_H = 2(1+\mu_D)G_H = 7320 \text{ MPa}, \tag{17}$$

$$\Psi = \frac{E_H}{E_D} = 0{,}791, \tag{18}$$

$$\varepsilon_E = \frac{f_{cS}}{E_H} = 0{,}00267. \tag{19}$$

The strain ε_E, shown in Figure 2 is in excellent agreement with the experimental peak strain shown in Figure 9 in [5]. The critical damage variable is D_1=0.55. The RDA efective stress is σ_{ef}=43.33 MPa. The static strength defined in [9] is f_{US2}=16.53 MPa. Table 1 shows the calculated model parameters taking into account the positive and negative Poisson's ratios.

Table 2: RDT model parameters for the cylinder made of permeable concrete

μ_1	0.275	μ_2	-0.379
$\varphi_1 = \frac{2\mu_D}{1-2\mu_D}$	1.222	$\varphi_2 = \frac{2\mu_2}{1-2\mu_2}$	-0.432
$E_1(0) = E_H(1+\varphi_1)$ [MPa]	16266.67	$E_2(0) = E_H(1+\varphi_2)$ [MPa]	4162.35
$K_{E1} = \frac{\varphi_1}{\sigma_{ef}}$	0.028205	$K_{E2} = \frac{\varphi_2}{\sigma_{ef}}$	-0.009955

Based on two Poisson's ratios the two stress-strain curves were defined in [9]:

$$\sigma_1 = \frac{1}{2K_{E1}}\left(\sqrt{1+4K_{E1}E_1(0)\varepsilon}-1\right), \tag{20}$$

$$\sigma_2 = \frac{1}{2K_{E2}}\left(\sqrt{1+4K_{E2}E_2(0)\varepsilon}-1\right). \tag{21}$$

The corresponding energy densities defined in [6] are:

$$W_{d1} = \frac{1}{2K_{E1}}\left[-\varepsilon_{cF} + \frac{-1+\left(1+4K_{E1}E_1(0)\varepsilon_{cF}\right)^{3/2}}{6K_{E1}E_1(0)}\right], \tag{22}$$

$$W_{d2} = \frac{1}{2K_{E2}}\left[-\varepsilon_{cF} + \frac{-1+\left(1+4K_{E2}E_1(0)\varepsilon_{cF}\right)^{3/2}}{6K_{E2}E_2(0)}\right], \tag{23}$$

where the ultimate strain is:

$$\varepsilon_{cF} = \frac{\sigma_{ef}}{E_H} = 0{,}00592. \tag{24}$$

The static strength is

$$f_{US2} = E_H\,\varepsilon_E\,\frac{1-\kappa_2}{\kappa_2}\sqrt{\frac{1}{3-4\kappa_2}} = 16{,}53 \text{ MPa} \tag{25}$$

where:

$$\kappa_2 = \left(\frac{\nu_T}{\nu_L}\right)^2 = 0{,}6375. \tag{26}$$

The measured compressive strength of 19,5 MPa lies above the static strength f_{US2}, indicating softening behavior. Figure 2 shows the calculated stresses and strains from the dynamic diagrams. Strain energy densities and porosities are respectively:

$$W_{el} = 0{,}12826 \text{ MJ/m}^3,$$

$$W_{d1} = 0{,}15178 \text{ MJ/m}^3,$$

$$W_{d2} = 0{,}09583 \text{ MJ/m}^3,$$

$$W_d = W_{d1} - W_{d2} = 0{,}05595 \text{ MJ/m}^3,$$

$$p_{max} = \frac{W_d}{W_{el}} = 0{,}436,$$

$$p_E = \frac{W_{d1} - W_{el}}{W_{el}} = 0{,}183,$$

$$p_f = \frac{2W_{el} - W_{d1} - W_{d2}}{W_d} = 0{,}159.$$

$$p_{min} = \frac{2W_{el} - W_{d1} - W_{d2}}{W_{el}} = 0{,}07.$$

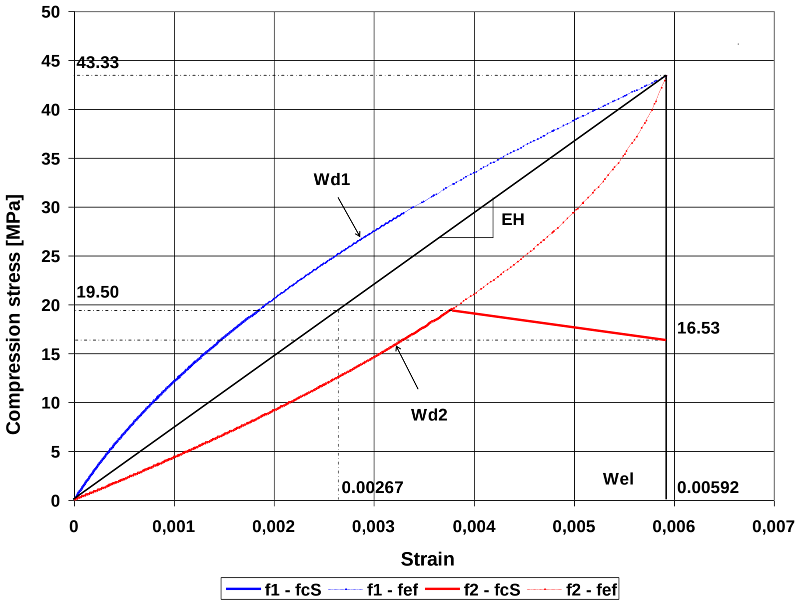

Figure 2: Dynamic stress-strain curves: with positive (blue line) and negative Poisson's ratio (red line) for the cylinder made of permeable concrete.

The maximum porosity of of 0,436 is completely in agreement with the design porosity p_{des}=0,436 [1]. The limit of measurable porosity of 0,183 is in excellent agreement with the effective design porosity p_{ef}=0,18 [1]. Finally, the porosity p_f controls the static strength f_{US2} as follows [6]:

$$\sigma\left(p_f\right)=\left\langle 1-\frac{p_f}{p_E}\left\{1-D_1\left[1-D\left(p_f\right)\right]\right\}\right\rangle \sigma_{ef} = 17{,}32 \sim 16{,}53 \text{ MPa}$$

where

$$D\left(p_f\right)=\left[\varphi_1-\frac{p_f\left(\varphi_1-\varphi_E\right)}{p_E}\right] \Big/ \left\{1+\left[\varphi_1-\frac{p_f\left(\varphi_1-\varphi_E\right)}{p_E}\right]\right\} = 0{,}4368.$$

Apart from the above parameters, the modulus of elasticity and density of the sample with zero porosity are necessary in this analysis. As defined in [6] they are respectively:

$$E_{H,0}=\frac{E_H}{1-p_f} = 8708275862 \text{ Pa},$$

$$\rho_0=\frac{\rho}{1-p_{min}} = 1611 \text{ kg/m}^3.$$

4.3 INTRINSIC PERMEABILITY BY RHEOLOGICAL DYNAMICS

The main goal in this section is to calculate the critical plastic shear viscosity $\eta_{N,cr}$ as defined in [10]. Required parameters for the analysed sample are:

$$k = E_H \frac{A_0}{L} = 143727864 \text{ N/m},$$

$$m = \rho V_0 = 0{,}294 \text{ kg},$$

$$T^D = \sqrt{\frac{m}{k}} = 0{,}000045253 \text{ s},$$

$$E_{K,cr} = \frac{E_H}{\varphi_1} = 5989090909 \text{ Pa},$$

$$\lambda_{K,cr} = E_{K,cr} T^D = 271023 \text{ Pa·s},$$

$$H'_{cr} = \frac{k\gamma}{E_{K,cr}} = 359{,}7 \text{ Pa},$$

$$\lambda_{N,cr} = H'_{cr} T^D = 0{,}016279 \text{ Pa·s},$$

$$\eta_{N,cr} = \frac{\lambda_{N,cr}}{3} = 0{,}005426 \text{ Pa·s}.$$

Comparing the plastic viscosity $\eta_{N,cr}$ with that found for different materials in [8], we see that it is close to the plastic viscosities of rock, not standard or recycled concrete. This shows that the plastic viscosity of porous concrete depends significantly on the aggregate, and not on cement and fine aggregates, which is the case with standard concrete. Therefore, the intrinsic permeability of the analysed pervious concrete is

$$k_{cr}\left(p_f\right) = \frac{\eta_{N,cr}}{\sqrt{E_{H,0}\left(1-\dfrac{p_f}{p_{\max}}\right)\rho_0\left(1-p_f\right)}} L = 1{,}98356 \cdot 10^{-10} \text{ m}^2.$$

Using Eq. (12), we obtain the water permeability coefficient K_w also known as hydraulic conductivity

$$K_w = \frac{k_{cr}\left(p_f\right)}{1.02 \cdot 10^{-7}} = \frac{1.98356 \cdot 10^{-10}}{1.02 \cdot 10^{-7}} = 0{,}00194 \text{ m/sec} = 1{,}94 \text{ mm/s}.$$

According to the results shown in Figure 4 in [1] for the porosity p_{ef}=0,16, the water-cement ratio was ~0,29. For this ratio, the measured water permeability coefficient was K_w~2 mm/s, which is in very good agreement with the calculated value in this paper.

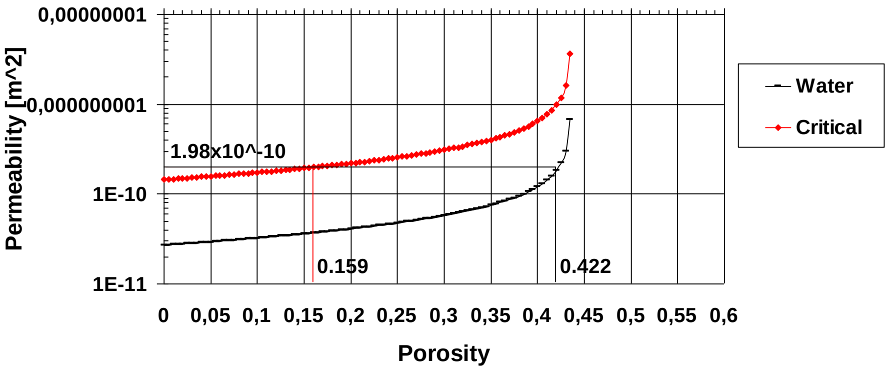

Figure 3: Permeability versus porosity for the analysed sample made of permeable concrete.

It is obvious that the experimental water permeability of the analysed sample is confirmed as the critical intrinsic permeability of solid calculated using RDT. According to Eq. (10) in which the viscosity of water is, we get the black line in Figure 3. Provided that the calculated value of the intrinsic permeability remains, the porosity threshold p_{th} is defined on the black line. In the analysed example, it is 0,422. The porosity threshold is the porosity at which the modulus of elasticity tends to zero [11]. It numerically approaches the maximum porosity of 0,436 defined by RDT. The porosity threshold can also be understood as the percolation threshold when liquid filtration through porous materials is established according to Darcy's law.

5 CONCLUSIONS

The critical intrinsic permeability defined in this paper depends only on the properties of the porous solid in the state of critical damping. In this work, the threshold of porosity is defined, which is a criterion for liquid flow.

As rheological dynamics explains, the fluid flow criterion is a state of critical damping of the solid with a porosity threshold tending to maximum porosity, when the solid can no longer resist fluid flow.

The presented analysis is valid in the case of a completely saturated porous medium, which is expected when Darcy's law is already applied. Given the unknowns related to open or closed pores (cracks) in materials, this analysis is useful in understanding the permeability of natural or artificial materials with cracking and eventual disintegration, but must contain new information about material behaviour, especially under natural conditions.

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