Међународна конференција
Савремена достигнућа у грађевинарству 24.-25. април 2014. Суботица, СРБИЈА

A 2-D SEDIMENT TRANSPORT MODEL FOR ALLUVIAL WATERCOURSES

Zoltan Horvat[1]

UDK: 627.157:519.6
DOI: 10.14415/konferencijaGFS2014.090

Summary: *This work presents the development and calibration of a two dimensional sediment transport and bed evolution model for alluvial watercourses that uses the active-layer and multiple size-class approach. Sediment mixtures are represented through a suitable (unlimited) number of size-classes, which can be subject to either suspended-load or bed-load transport (or both), depending on local hydraulic conditions. The governing depth-averaged sediment transport equations are given in orthogonal curvilinear coordinates and solved using the fractional step method, which resulted in two successive steps (advection and diffusion). The developed model is assessed using field measurements conducted on the Danube River located in the border area between Hungary and Serbia. Analysis of the measured and computed values confirmed the developed model's reliability.*

Keywords: *Numerical model, sediment transport equations, field measurements*

1. INTRODUCTION

The sediment transport in alluvial rivers has direct influences on the evolution of rivers, banks and associated aquatic habitat. Two dimensional depth averaged sediment models are widely used [1,2] since they are more detailed than 1-D models, and much faster than 3-D models. Since natural watercourses contain non-uniform sediment mixtures, using the active-layer and multiple size-class approach would be reasonable [3].

The split-operator approach, when applied to sediment transport, typically results in a two step solution, with separate advection and diffusion step [4]. Using this method provides the possibility to treat each of the obtained steps with the most appropriate numerical method. The objective of this paper is to present a developed 2-D depth averaged model and its implementation and assessment using field measurements.

2. MODEL FORMULATION

Modeling the suspended sediment transport with the advection-diffusion equation in addition of the fall velocity term is a well established procedure. For bed and near-bed processes the active-layer concept was adopted [3], coupled with a modeling approach

---

[1] Zoltan Horvat, dipl.inž. građ., University of Novi Sad, Faculty of Civil Engineering Subotica, Kozaračka 2a, Subotica, Serbia, tel: +381 24 554 300, e-mail: horvatz@gf.uns.ac.rs

that divides the sediment mixture into a suitable number of sediment size-classes, *k*=1,2,…,*K*. The split-operator approach breaks up the suspended sediment mass conservation equation in two successive steps, the advection and diffusion step. The advection step equation (1), marked with upper index *a*, is coupled with the mass conservation equation for *k*-th size class of active-layer sediment, equation (2), and the global mass conservation equation for bed sediment, equation (3),

$$\left(\frac{\partial C_k}{\partial t}\right)^a = -\frac{u}{h_\xi}\frac{\partial C_k}{\partial \xi} - \frac{v}{h_\eta}\frac{\partial C_k}{\partial \eta} + \frac{S_k}{\rho d} \tag{1}$$

$$\rho_s\left(1-p\right)\frac{\partial\left(\beta_k E_m\right)}{\partial t} + \frac{1}{h_\xi h_\eta}\frac{\partial}{\partial \xi}\left[h_\eta\left(q_\xi\right)_k\right] + \frac{1}{h_\xi h_\eta}\frac{\partial}{\partial \eta}\left[h_\xi\left(q_\eta\right)_k\right] = -S_k + \left(S_f\right)_k \tag{2}$$

$$\rho_s\left(1-p\right)\frac{\partial z_b}{\partial t} + \sum_{k=1}^{K}\left\{\frac{1}{h_\xi h_\eta}\frac{\partial}{\partial \xi}\left[h_\eta\left(q_\xi\right)_k\right] + \frac{1}{h_\xi h_\eta}\frac{\partial}{\partial \eta}\left[h_\xi\left(q_\eta\right)_k\right]\right\} = -\sum_{k=1}^{K} S_k \tag{3}$$

In equations (1), (2) and (3) C_k denotes the dimensionless suspended sediment concentration, ρ_s is the sediment density, ρ the water-sediment mixture density, *p* the bed material porosity, q_k the bed-load flux, S_k is the source term, while β_k, E_m and $(S_f)_k$ respectively denote the active-layer size fraction, active-layer thickness and floor source term. After acquiring the simultaneous solution for equations (1), (2) and (3), the diffusion equation (4), marked with upper index *d*, is to be solved,

$$\left(\frac{\partial C_k}{\partial t}\right)^d - \left(\frac{\partial C_k}{\partial t}\right)^a = \frac{1}{h_\xi h_\eta d}\frac{\partial}{\partial \xi}\left[\nu_{\text{sed}}\frac{h_\eta}{h_\xi}\frac{\partial C_k}{\partial \xi}d\right] + \frac{1}{h_\xi h_\eta d}\frac{\partial}{\partial \eta}\left[\nu_{\text{sed}}\frac{h_\xi}{h_\eta}\frac{\partial C_k}{\partial \eta}d\right] \tag{4}$$

where ν_{sed} is the turbulent diffusion and dispersion coefficient.
The advection is solved using the characteristic method that transforms Eq. (1) into

$$\frac{DC_k}{Dt} = \frac{S_k}{\rho d} \tag{5}$$

along the characteristic curve (trajectory) defined by

$$\frac{d\xi}{dt} = \frac{u}{h_\xi}, \quad \frac{d\eta}{dt} = \frac{v}{h_\eta}. \tag{6}$$

In order to shorten the computation time, Eqs. (6) are solved in a way that allows the characteristic curve to stretch through multiple computational cells. This is achieved by developing an algorithm that divides the considered trajectory into an arbitrary number of straight segments, l=1, 2,..., L, each bounded on both ends by neighboring computational cells. Finally, integration of Eq. (5) along the trajectory yields the suspended sediment concentration equation. The size-class specific Eqs. (2) and the global mass conservation Eq. (3) are discretized by integrating them over the time step and the control volume built around a main computational point. The flow computations furnish the necessary velocity at control volume faces, while the numerical treatment of bed-load flux is equivalent to an upwind scheme with explicitly expressed sediment variables. The discretized Eqs. (1), (2) and (3) are coupled and they need to be solved simultaneously. The total number of equations is 1+2K, while there are 2+6K unknowns, hence the need for a system closure that is provided by empirical equations.

The active layer thickness is related to the erosion intensity [1]. The bed-load flux is computed using the empirical equation proposed by van Rijn [5], supplemented with the allocation parameter γ, hiding factor ζ and size-class distribution in the active layer β. The source term through the active layer floor is given by Refs.[1,3]. Finally, the source term is defined as the difference between entrainment and deposition. The entrainment term is modeled employing the same principles as for the diffusion flux [6],

$$E_k^{\text{sed}} = -\varepsilon_k \left(a_k + \Delta a\right) \frac{\left(\rho C_k\right)_{a_k+\Delta a} - \left(\rho C_k\right)_{a_k}}{\Delta a}, \tag{7}$$

where ε_k is the diffusion coefficient, while a_k and Δa are calibration parameters. Parameter a_k presents the distance from the bed level, where the entrainment flux is determined, and Δa is the distance used to compute the concentration gradient. The deposition is evaluated at distance a_k+Δa from the bed level as

$$D_k^{\text{sed}} = \left(w_k \, \rho C_k\right)_{a_k+\Delta a}, \tag{8}$$

where w_k denotes the fall velocity.

The suspended sediment diffusion it is numerically benign, therefore is given merely as a short summary. Equation (4) is discretized with the Crank-Nicolson scheme, divided in two directions using the alternating direction implicit (ADI) method [7], and solved using the double-sweep algorithm [8].

3. FIELD MEASUREMENTS

Field measurements were conducted on a site sight bounded by Bezdan (rkm 1425.5) in Serbia and Mohacs (rkm 1446.9) in Hungary. Within this reach seven data ranges were selected for detailed flow field and sediment data measurements (Fig. 1). These ranges were placed between rkm 1438 and rkm 1432, at 1 km apart. The data collection campaign took place during five days (23-27 May 2011). Each range had seven verticals, where velocity distribution profiles were collected with the use of ADCP. Out of these seven verticals, five were chosen for detailed sediment data collection that consisted of suspended and bed sediment sampling. Suspended sediment data was collected at five points on any given vertical. Sample processing provided sediment size-class distributions needed for the developed numerical model.

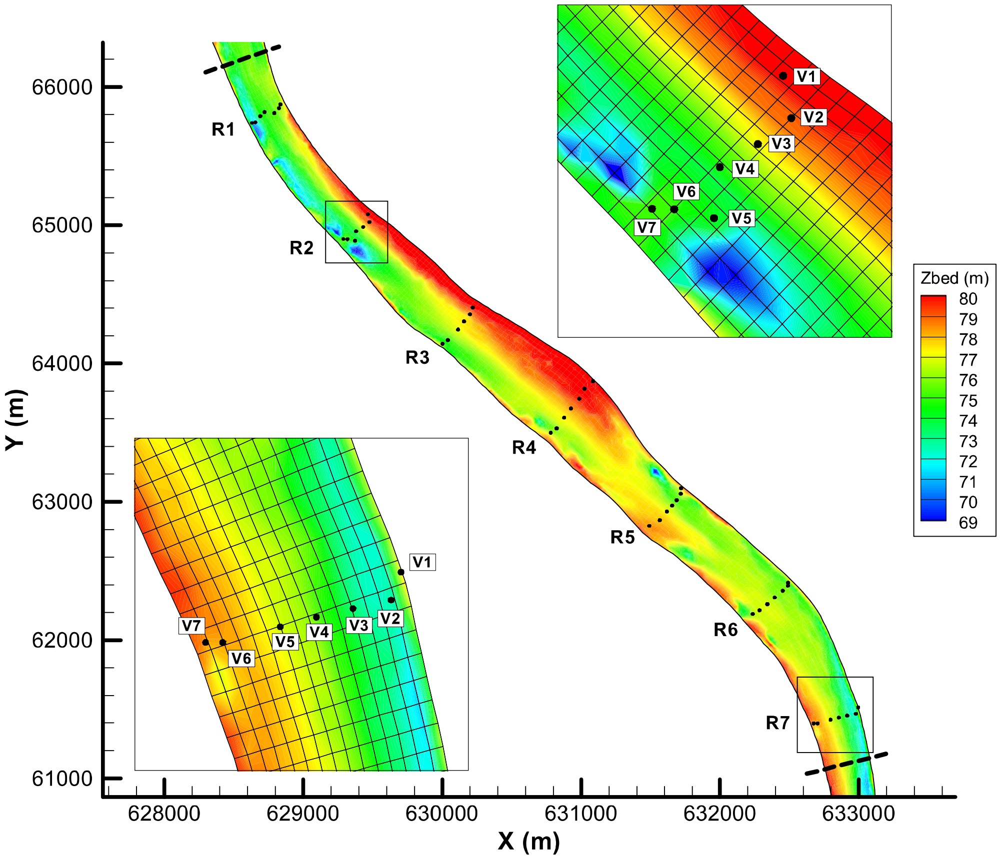

Figure 1. Selected data ranges

4. RESULTS

The sediment mixture was divided into 10 size-classes given in Table 1.

D_k (mm)	0.00224	0.00707	0.0141	0.0316	0.0707	0.112	0.177	0.354	0.707	4.0
k	1	2	3	4	5	6	7	8	9	10

Table 1. Characteristic diameters and sediment size-classes

The simulation time was five days, with two additional days that were allocated as the stabilization period. Initial conditions for suspended and bed sediment were obtained by averaging the appropriate measured values. The upstream boundary condition for the suspended sediment is the averaged value of the measured concentrations in the first data range. Bed material boundary conditions were treated the same way. The improved sediment advection algorithm enabled the selection of a 90sec time step that produced the longest trajectory consisting of six segments. Calibration of the sediment model was done by altering a and Δa in Eq. (7-8). In the sediment computation, the continuity equation error was monitored separately for each size-class. The largest relative error for a single size-class stayed under 0.2% (8th size-class), while the cumulative value of the relative error for all size-classes did not exceed 0.3%. Figure 2 depicts the concentrations for each size-class, while the results on Fig. 3 present overall concentrations for ranges 4 and 6 computed with the model using multiple sediment size-classes approach.

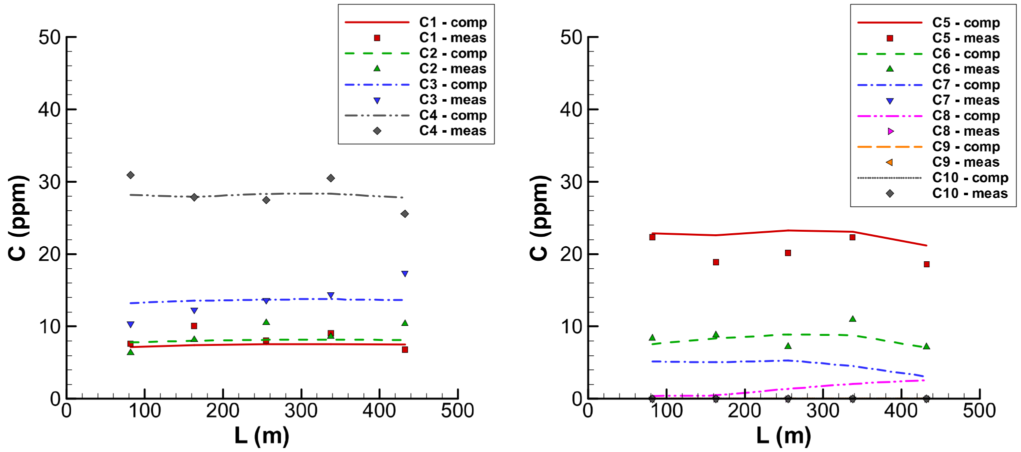

Figure 2. Suspended sediment concentration for k= 1-4 and k= 5-10 at data range 4

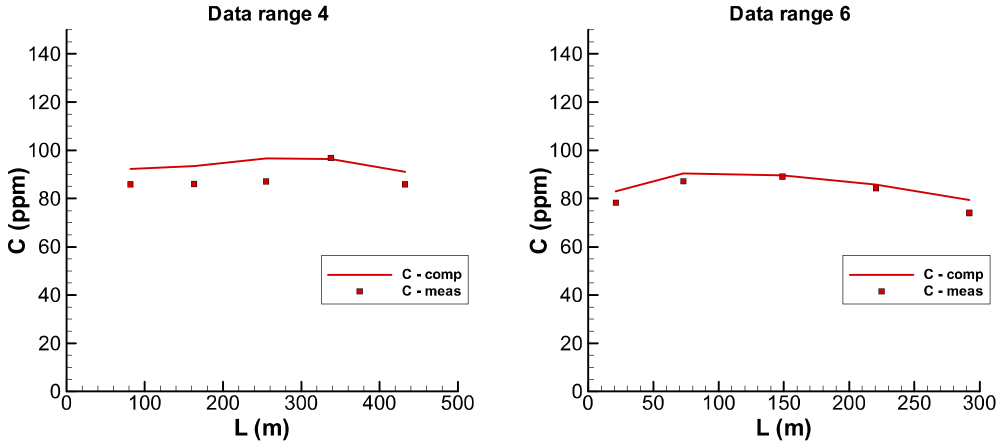

Figure 3. Total suspended sediment concentration at data ranges 4 and 6

5. CONCLUSION

A 2-D (depth-averaged) numerical model, simulating water flow, sediment transport and bed evolution was developed. The model implements the split-operator approach, allowing separate treatment of the troublesome advection terms. The advection step was solved using the improved algorithm for the characteristic method allowing trajectories to extend through multiple computational cells. The incorporated sediment model employs the active-layer concept. The sediment mixture was divided into size-classes. Since the simulation results show good agreement with the measurements, and the continuity equation error was negligible, the presented model is suitable for application in natural watercourses.

ACKNOWLEDGEMENTS

This work was funded by the Ministry of Education and Science of the Republic of Serbia, project number TR 37009.

REFERENCES

[1] Hung, M.C., Hsieh, T.Y., Wu, C.H., Yang, J.C.: Two-Dimensional Nonequlibrium Noncohesive and Cohesive Sediment Transport Model. *Journal of Hydraulic Engineering*, **2009.**, vol. 135, № 5, p.p. 339-382.

[2] Zhou, G., Wang, H., Shao, X., Jia, D.: Numerical Model for Sediment Transport and Bed Degradation in the Yangtze Channel Downstream of Three Gorges Reservoir. *Journal of Hydraulic Engineering*, **2009.**, vol. 135, № 9, p.p. 729-740.

[3] Spasojevic, M., Holly, F.M.Jr.: 2-D bed evolution in natural watercourses - New simulation approach. *Journal of Waterway, Port, Coastal, and Ocean Engineering*, **1990.**, vol. 116, № 4, p.p. 425-443.

[4] Isic, M., Horvat, Z., Spasojevic, M.: Advection step in the split-operator approach applied to river modeling. *Applied Numerical Mathematics*, **2013.**, vol. 72, p.p.1-18.

[5] Van Rijn, L.C.: Sediment Transport, Part I: Bed Load Transport. *Journal of Hydraulic Engineering*, **1984.**, vol. 110, № 10, p.p. 1431-1456.

[6] Wu, W.: *Computational River Dynamics*, Taylor & Francis Group, London, **2008**.

[7] Hsieh, T.Y., Yang, J.C.: Implicit Two-Step Split-Operator Approach for Modelling Two-Dimensional Open Channel Flow. *Journal of Hydroscience and Hydraulic Engineering*, **2004.**, vol. 22, № 2, p.p. 113-139.

[8] Hirsch, C.: *Numerical Computation of Internal and External Flows – Volume 1 – Second edition,* Elsevier, Oxford, **2007**.

РАВАНСКИ МОДЕЛ ТРАНСПОРТА НАНОСА У АЛУВИЈАЛНИМ ВОДОТОЦИМА

Резиме: *Овај рад представља развој и калибрацију раванског модела транспорта наноса и деформације корита у алувијалним водотоцима, који користи концепт* *активног слоја и дискретних гранулометријских интервала. Мешавина наноса се моделише преко изабраног (неограниченог) броја дискретних гранулометријских интервала, при чему се наносни материјал једног оваквог интервала, у зависности од локалних хидрауличких услова, може кретати у виду суспендованог или вученог наноса (или на оба начина). Једначине транспорта наноса осредњене по дубини тока су дате у ортогоналном криволинијском координатном систему, након чега су решаване применом методе разломљених корака, која даје два рачунска корака (адвективни и дифузиони). Тачност развијеног нумеричког модела је процењена користећи мерења спроведена на деоници реке Дунав у пограничној области између Мађарске и Србије. Упоређивањем мерених и срачунатих вредности наносних величина установљена је поузданост развијеног нумеричког модела.*

Кључне речи: *Нумерички модел, једначине транспорта наноса, теренска мерења*