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# SEMI-RIGID FIXED END BEAMS STABILITY

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***Summary:*** *Joint rigidity influence on static reinforced-concrete slabs design the paper presents the influence calculation method in the beams with flexible or semi-rigid fixed ends. The value of the fixed end degree can be determined only experimentally and the above mentioned values can be estimated only arithmetically. Numerical examples indicate the change in the static influence in the beams, depending upon the fixed end degree ie joint rigidity degree.*

***Keywords:*** *Elastic, rigid and semi-rigid fixed end, degrees of rigidity, degrees of connection stiffness*

## 1. INTRODUCTION

The paper presents the theoretical analysis of the beams loaded by pressing forces at the simply supported and free edges of the ones. However, in the real structures it is difficult to accomplish the complete fixity as well as the simple support, so the side of the beam is usually partially semirigidly connected in joints, ie elastically fixed. The fixing degree is represented by the full fixing and actual fixing moment ratio MlMo. Fixing degree values can be determined only experimentally while arithmetically the ones can be estimated only by assuming that they vary within certain empirically detemlined limits. The fixing degree estimate ie MlMo ratio is extremely important during the statistical calculation of the structures, taking into consideration the fixing degree influence on the structure behavior while in operation. Consideration of the fixed end rigidity problem in the surface girders is more complex than the one in the linear ones, for the surface

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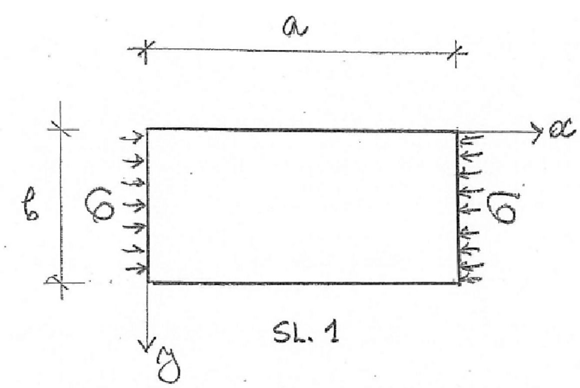
girders torque influence affects the deflection and bending moment size much more significantly. Plate bending problem occurs, for example, in buildings with mezzanine floor-ceiling structures. In order to accomplish the rigid fixed end the girder, or the wall in these structures, which the plate of the ceiling is fixed onto, should be absolutely rigid, and in reality it is not the case. The behavior of the joints between the ceilings themselves as well as the ceilings and the pillars, and therefore the behavior of the whole structure, depends directiy upon the corresponding joints rigidity, so the greatest attention should be paid to this issue, during the prefabricated construction. If we take into consideration the change of the basic dynamic characteristics ofthe structure, the joint rigidity ie joints have a large influence on the structural analysis and they are often quite crucial during the dynamic ones. Therefore, the elastic thin plates, as the ones commonly used in the construction industry, are being taken into consideration. For example, the steel plates on the factory Tigar in Pirot have been taken as an illustration.

## 2. EVALUATION OF CRITICAL STRESS FOR PLATES

As for the plates under load, uniformly positioned along the edges, x=0 and x=b (Fig. 1) the critical voltage is calculated according to the following formula:

$$\sigma kv = k \cdot \pi^2 \cdot D/b^2 \cdot h = k \cdot \pi^2 \cdot D/12(1-\upsilon^2) \cdot (h/b)^2 \quad (1)$$

The coefficient k depends on the boundary conditions and the side panels length ratio
(a,b) = rectangular plate sides
h = plate thickness
E,G = elasticity material modulus during stretching, compression and rigidity
$\upsilon$ = Poisson's ratio
D = E $h^3/12(1-\upsilon^2)$ - cylindrical rigidity:




SL. 1

Kritični napon $(\sigma_{x\varrho})_{kr}$ iznalazi se iz obrasca (: :.1). Vrednosti koeficijenta $K$, za slučaj da su sve četiri ivice ploče zglobno oslonjene, date su u tablici 1.

Tablica ! ;.1

Vrednosti koeficijenta $K$ u obrascu ( ;.1)

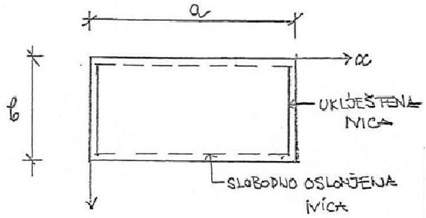
| Granični uslovi | a/b | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1,0 | 1,1 | 1,2 | 1,3 | 1,4 | 1,5 | 1,6 | 1,7 | 1,8 | Od 1,9 do ∞ |
| Sve ivice ploče zglobno oslonjene | 27 | 13,2 | 8,41 | 6,25 | 5,14 | 4,53 | 4,2 | 4,04 | 4,0 | 4,04 | 4,13 | 4,28 | 4,47 | 4,34 | 4,2 | 4,08 | 4,05 | 4,0 |

ZA SLUČAJ UKLJEŠTENJA PO IVICAMA y=0 i y=b a DRUGE DVE IVICE ZGLOBNO OSLONJENE VREDNOSTI KOEFICIJENTA K SU DATE U TABELI 2. (SL.2)

Tablica ! 2

Vrednosti koeficijenta $K$ u obrascu ( ;.1)

| Granični uslovi | a/b | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0,6 | 0,8 | 1,0 | 1,2 | 1,4 | 1,6 | 1,8 | 2,0 | 3,0 | ∞ |
| Uklještenje po ivicama $y = 0$ i $y = b$ a druge dve ivice zglobno oslonjene | 7,05 | 7,29 | 7,69 | 7,15 | 7,04 | 7,2 | 7,05 | 7,0 | 7,15 | 7,0 |




SL.2

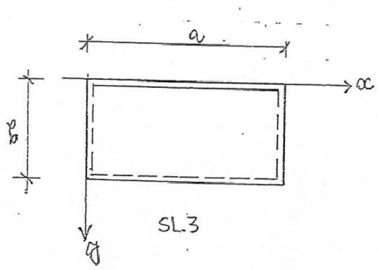
Coefficient k value in the case of the fixed end edge y=0 and the ones of the same edges hinge joint supported (Fig.3) are given in the Table 3.

Tablica : .3

Vrednost koeficijenta $K$ u obrascu (:.³1) za slučaj uklještenja ivice $y = 0$ i zglobnog oslanjanja svih ostalih ivica

| a/b | $GI_k/Db$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0,25 | 0,5 | 1,0 | 2,0 | 4,0 | 6,0 | 10,0 |
| 0,5 | 6,52 | 6.63 | 6,71 | 6,77 | 6,81 | 6,82 | 6,83 |
| 0,6 | 5,44 | 5,58 | 5,70 | 5,79 | 5,85 | 5,87 | 5,89 |
| 0,7 | 4,85 | 5,03 | 5,19 | 5,32 | 5,40 | 5,44 | 5,46 |
| 0,8 | 4,56 | 4,75 | 4,96 | 5,13 | 5,25 | 5,30 | 5,34 |
| 0,9 | 4,42 | 4.64 | 4,89 | 5,11 | 5,28 | 5,40 | 5,40 |
| 1,0 | 4,39 | 4,64 | 4,93 | 5,21 | 5,43 | 5,52 | 5,60 |

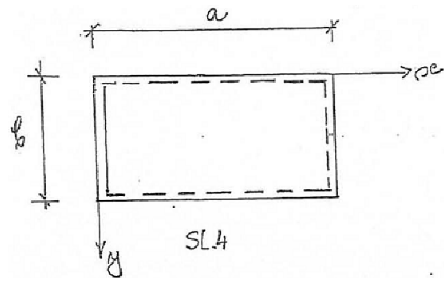
$GI_k$ — krutost prema uvrtanju rebra koje se pripaja uklještenom kraju (karakteriše stepen uklještenja).




SL.3

## 3. THE CASE OF THE ELASTIC FIXED END

Coefficient k value in the case ofthe elastically fixed end edge is x=0, while the other edges are hinge joint supported (Fig 4.) are all given in the Table 4.



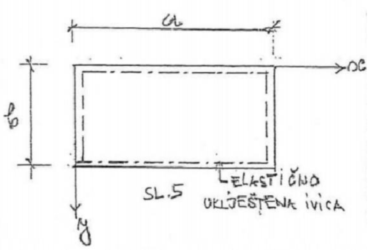
SL.4

As for the semi-rigid fixed end edges y=0, y=a whereas the other two sides are hinge joint supported, the coefficient k values are given in the Tables 5 and 6 (Fig. 5)

Tablica .4

Vrednosti koeficijenta $K$ u obrascu (18.1) u slučaju elastičnog uklještenja ivice $x = 0$, dok su ostale ivice zglobno oslonjene

| $\frac{a}{b}$ | $GI_k/Da$ | | | | | |
|---|---|---|---|---|---|---|
| | 0,25 | 0,5 | 1,0 | 2,0 | 4,0 | 10 |
| 0,33 | 4,08 | 4,09 | 4,10 | 4,11 | 4,11 | 4,11 |
| 0,5 | 4,16 | 4,19 | 4,21 | 4,23 | 4,23 | 4,24 |
| 1,0 | 4,32 | 4,46 | 4,60 | 4,70 | 4,77 | 4,82 |
| 1,5 | 5,10 | 5,39 | 5,76 | 6,14 | 6,48 | 6,73 |
| 2,0 | 6,70 | 7,06 | 7,60 | 8,28 | 8,98 | 9,68 |
| 2,5 | 8,87 | 9,28 | 9,94 | 10,89 | 12,00 | 14,75 |
| 3,0 | 11,59 | 12,02 | 12,77 | 13,91 | 15,43 | 17,50 |



SL.5

Tablica .5

Vrednosti koeficijenta $K$ u obrascu ( .1) u slučaju da su podjednako elastično uklještene ivice $y = 0$ i $y = b$, dok su preostale dve ivice zglobno oslonjene

| $\frac{a}{b}$ | $GI_k/Da$ | | | | | |
|---|---|---|---|---|---|---|
| | 0,25 | 0,5 | 1,0 | 2,0 | 4,0 | 10,0 |
| 0,5 | 6,84 | 7,09 | 7,31 | 7,47 | 7,57 | 7,64 |
| 0,6 | 5,80 | 6,12 | 6,43 | 6 68 | 6,85 | 6,96 |
| 0,7 | 5 24 | 5,64 | 6,06 | 6,42 | 6,67 | 6,86 |
| 0,8 | 4,96 | 5,42 | 5,97 | 6,44 | 6,80 | 7,08 |
| 0,9 | 4,84 | 5.36 | 5,99 | 6,62 | 7,12 | 7,52 |
| 1,0 | 4,79 | 5,39 | 6,14 | 6,92 | 7,57 | 7,64 |

Tablica 1

Vrednosti koeficijenta $K$ u obrascu ( 1) u slučaju elastičnog uklještenja ivica $y = 0$, i $y = b$, dok su druge dve ivice zglobno oslonjene

| $\alpha$ | $\frac{a}{b}$ | $GI_k/Db$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | 0,25 | 0,50 | 1,0 | 2,0 | 4,0 | 10,0 |
| 1,0 | 0,5 | 12,83 | 13,37 | 13,86 | 14,23 | 14,46 | 14,62 |
| | 0,6 | 11,03 | 11,69 | 12,34 | 12,87 | 13,23 | 13,48 |
| | 0,7 | 10,22 | 10,86 | 11,72 | 12,44 | 12,96 | 13,35 |
| | 0,8 | 9,60 | 10,50 | 11,55 | 12,53 | 13,26 | 13,83 |
| | 0,9 | 9,41 | 10,42 | 11,68 | 12,94 | 13,93 | 14,74 |
| | 1,0 | 9,40 | 10,52 | 11,99 | 13,54 | 14,46 | 14,62 |
| 2,0 | 0,4 | 34,05 | 36,04 | 37,79 | 39,03 | 39,80 | 40,32 |
| | 0,5 | 30,45 | 32,83 | 35,17 | 37,01 | 38,24 | 39,10 |
| | 0,6 | 29,11 | 31,90 | 34,92 | 37,52 | 39,35 | 40,72 |
| | 0,7 | 29,00 | 32,17 | 35,91 | 39,41 | 42,06 | 44,14 |

*Napomena.* Vrednost $\alpha=0$ odnosi se na slučaj da je pritiskujuća sila ravnomerno raspodeljena; $\alpha=2$ važi pri čistom savijanju; ako je $\alpha<2$ onda je po sredi kombinacija savijanja i pritiska, a ako je $\alpha>2$ onda kombinacija savijanja i zatezanja.

## 4. NUMERICAL EXAMPLES

1. For the rectangular hinge joint supported plate whose side lengths are a=b=6.0m, ie a) a/b=1 and the case of b) a/b=1.5, plate thickness h=0.01m, $E = 2.1\times10^8$ KN/m$^2$ and the calculated critical voltage is $\upsilon$=0.30 I

**Solution**: Formula (1) and Table 1 is applied

a) $\sigma_{kv} = k\cdot\pi^2\cdot D/12(1-\upsilon^2) \cdot (h/b)^2 = 4/9{,}8696\cdot2{,}1\cdot10^8/12(1-0{,}3^2)(0{,}01/6)^2 = 4\cdot3{,}1089\cdot10^8/10{,}92\cdot4\cdot189800000\cdot0{,}00000278 = 4\cdot527{,}074 = 2108{,}30$ kN/m$^2$

b) $\sigma_{kv} = k\cdot\pi^2\cdot D/12(1-\upsilon^2) \cdot (h/b)^2 = k\cdot9{,}8696\cdot2{,}1\cdot10^8/12(1-0{,}3^2)(0{,}01/4)^2 = 4{,}34\cdot189800000\cdot0{,}00000277 = 4{,}34\cdot5257{,}746 = 2281{,}737$ kN/m$^2$ $=228.1$ t/m$^2$ $=22.81$ kg/cm$^2$

2. Calculated critical voltage (Formula 1) in the case that the edges are uniformly rigidly joined y=0 and y=b, while the remaining two edges are hinge joint supported.

Sides a=b=4m, h=0,015m, E=2,1·10$^8$KN/m$^2$, $\upsilon$=0,3

a) a/b=1,0 b) a/b=0,5 for G·Ix/D·a=0,25; 1,0 and 10

**Solution**: Formula (1) and Table 5 is applied

a) $\sigma_{kv}$ 0,25= $k\cdot\pi^2\cdot D/12(1-\upsilon^2) \cdot (h/b)^2 = 4{,}79/9.8696\cdot2{,}1\cdot10^8/12(1-0{,}3^2)(h/4)^2 = 4{,}79\cdot189800000(0.015/4)^2 = 4.79\cdot18980\cdot1.406 = 4.79\cdot2669.06 = 12784{,}81$ kN/m$^2$ = 127,85kN/cm$^2$

$\sigma_{kv}$ 1,0 $=6{,}14\cdot2669{,}06=16388{,}028$ kN/m$^2$ = 163,88kN/cm$^2$

$\sigma_{kv}$ 10 $=7{,}64\cdot2669{,}06=20391.162$ kN/m$^2$ = 203,91kN/cm$^2$

b) $\sigma_{kv}$ 0,25= k·π²·D/12(1-υ²)·(h/b)²=4,79·26987187,5(0,015/6)= 4,79·189800000·0,00000625=4,79·1186,25=5682,2 kN/m²=56,82kg/cm²
$\sigma_{kv}$ 1,0=6,19·1186,25= 7283,57kN/m²=72,84kg/cm²
$\sigma_{kv}$ 10=7,64·1186,25= 9062,95kN/m²=90,63kg/cm²

3. Calculated critical voltage (Formula 1) in the case of elastically fixed end edge y=O and y=b, while the other two edges are crank supported.

3.1. Sides a=b=4,0m, h=0,015m, E=2,1·10⁸KN/m², υ=0,3 and
a) a/b=1,0 and b) a/b=0,5 for G·Ix/D·a=0,25; 1,0 and 10; α=1,0.
3.2. Sides a=b=4,0m, h=0,015m and side ratio
a) a/b=0,7 and b) a/b=0,4 for G·Ix/D·a=0,25; 1,0 and 10; α=2,0.
5,71 10

**Solution**: Formula (1) and Table 6 is applied
3.1 a) $\sigma_{kv}$ 0,25= k·π²·E/12(1-υ²)·(h/b)²=9,40·189800000· (0,015/5,71)²
$\sigma_{kv}$ 1,0 = 11,99·1309,804=15709,570KN/m²=157,02kg/cm²
$\sigma_{kv}$ 10 = 14,62·1309,804=19149,34KN/m²=191,49kg/cm²
b) $\sigma_{kv}$ 0,25=12,83·189800000·0,00000225=12,83·127,05=5479,05kN/m² =54,79kg/cm²
$\sigma_{kv}$ 1,0 = 13,86·127,05=5918,91kN/m²=59,19kg/cm²
$\sigma_{kv}$ 10 = 14,62·427,05=6243,47kN/m²=62,43kg/cm²
3.2 a) $\sigma_{kv}$ = k·π²·E/12(1-υ²)·(h/b)²=k·9,8696·2,1·10⁸/10,52·(0,015/5,71)²= = k·1309,804
$\sigma_{kv}$ 0,25=29·1309,804= 37984,316kN/m²=379,84kg/cm²
$\sigma_{kv}$ 1,0 = 35,95·1309,804=47035,01kN/m²=470,35kg/cm²
$\sigma_{kv}$ 10 = 44,14·1309,804=57814,748kN/m²=578,15kg/cm²
b) $\sigma_{kv}$ = k·π²·E/12(1-υ²)·(h/b)²=k·9,8696·2,1·10⁸/10,92·(0,015/10)²= = k·427,05
$\sigma_{kv}$ 0,25=34,05·427,05=14541,052kN/m²=145,41kg/cm²
$\sigma_{kv}$ 1,0 = 37,79·427,05=16138,22kN/m²=161,38kg/cm²
$\sigma_{kv}$ 10 = 40,32·427,05=17218,65kN/m²=172,18kg/cm²

## 5. CONCLUSION

On the basis of the performed numerical analysis on the rectangular plates of different sizes with different boundary conditions and elastical rigid joint, interesting conclusions could be drawn. There is a clear indicator that the critical voltage is higher in the smaller plate sizes and the lowest voltage is in the hinge joint supported plates.
The critical voltage is significantly higher in the case ofthe semi-rigid (elastically rigid joint) plate and it is even greater when the value G·Ix is higher ie the rigidity in accordance with the rib twisting (it is the case of the stability loss) that is attached to the fixed end, where the dimension of G·Ix characterizes the degree of fixing.

In addition, the critical voltage is also significantly higher, if the value is $\alpha=2$, which applies in the case of pure bending. As for $\alpha<2$ it is the combination of bending and compression, and if the value is $\alpha>2$ then it is the combination of bending and tension.

## ACKNOWLEDGEMENT

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# СТАБИЛНОСТ ПОЛУКРУТО УКЉЕШТЕНИХ ПЛОЧА

***Резиме:*** *У раду се даје један метод за срачунавање утицаја код плоча са еластичним или полукрутим укљуштењима. Вредности степена укљуштења могу се одредити једино експерименталним путем, док се рачунским путем могу само проценити те вредности. На нумеричким примерима се указује па промену статичких утицаја код плоча у зависности од степена укљештења тј. степена крутости веза.*

***Кључне речи:*** *Еластично, круто и полукруто укљештење, степен укљештења, степен крутости везе*