

## STATISTICAL PREDICTIVE MODEL FOR HORIZONTAL DISPLACEMENTS OF “VRUTCI” DAM

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**Summary:** *This paper presents a development of statistical predictive model for relative horizontal displacements of “Vrutci” dam in function of air temperature and reservoir water level, as reversible effects and in function of time passed since the beginning of dam exploitation, which takes irreversible effects into account. Some parts of technical monitoring system, which are important for the model development, are described, with special attention paid to their behaviour. Measurements of horizontal radial displacements of the dam, air temperature and reservoir water levels, which all present data series suitable for regression analysis, are presented. For statistical analysis and coefficient defining in predictor functions, software “Expert Tool” developed by Water Institute “Jaroslav Černi” was employed. A data series with expected measurement values is formed and its level of agreement with actual measured data is determined. The effects of input variables on the predictive model formulation are discussed.*

**Key words:** *displacements, level, temperature, regression, model.*

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## 1. INTRODUCTION

The "Vrutci" dam was built at the entrance to the river Đetinja canyon, 12.0 km upstream from Užice and it has been exploited since 1984. (Figure 1.).

The dam reservoir performs long-term water levelling and has a multipurpose character with two basic tasks, the first of which is water supply to the city of Užice and town Sevojno and the second - transformation of big water waves of the basin towards the dam profile. Its influence on small waters refining is also significant.

The dam belongs to the category of high dams and as such must be a subject of technical monitoring. It is known that dams are structures whose potential damage or failure can have serious consequences. Within the dam maintenance, setting up, maintaining and application of an appropriate technical monitoring system is essential, as it provides a series of data relevant for dam monitoring. At the time of construction, an adequate system for technical monitoring of the dam was designed and implemented.



*Figure 1. 'Vrutci' dam*

## 2. DAM STRUCTURE DESCRIPTION

"Vrutci" dam is an arch dam with double curvature, 77.0 m high, with a crest length of 241.0 m (Figures 2 and 3). The thickness of the arch at the bottom is 10.71 m, and at the crest 3.01 m.

The dam is divided into 15 sections with dilatation joints, which were closed with plastic dilatation strips on upstream and downstream faces, and injected after completion of concreting.



On the downstream face of the dam, revision passageways and access to outlet gates were made. Two higher passageways at 610.0 and 589.20 m were made through the length of the dam, while the lowest passageway at 574.20 m is located on the left half of the dam and it leads to the outlet gate.

Turning the river during construction was done through an abandoned tunnel of a narrow-gauge railway.

The dam has 83200.0 m<sup>3</sup> of concrete built into it, and concrete works have been completed in three construction seasons.

Over the crest of the dam and the spillway, 3.60 m wide bridge for the road traffic was constructed.

For large waters evacuation, the spillway, which is located on the crest of the dam, is constructed in the central part. Spillway elevation is 627.0 m, which is almost 6.0 meters higher than the normal water surface (621.30 m). The spillway consists of 7 spillway parts, each with length of 6.80 m, so that the total effective spillway length is 47.60 m. The spillway length with the bridge columns is 48.80 m. The spillway is free, without water gates.

On the "Vrutci" dam, there are four outlets whose total capacity (maximum flow) is 136.0 m<sup>3</sup>/s. Two outlets are located at 575.0 m and two are located at 590.0 m (the axis of the outlets). Both gates are located on the downstream face in the gate building.

Stilling pool of "Vrutci" dam has a function of receiving and settling the water flowing over the top and the water passing through the outlets. The stilling pool and the slope are covered with reinforced concrete slab. The slab at the bottom of the stilling pool is 0.80 m thick, and on the slopes, it has a thickness of 1.0 m. The slab is separated by expansion joints and anchored with profiled rebars.

### 3. DESCRIPTION OF TECHNICAL MONITORING SYSTEM PARTS RELEVANT FOR STATISTICAL MODELLING

The water level in the reservoir is measured by a staff gauge located on the left slope of the reservoir [1].

Air temperature is measured at three locations. Two thermometers are located on the downstream face of the section 8 at 610.0 and 590.0 m. These thermometers are equipped with sensors that work on principle of electrical resistance and are telemetrically read at the measuring station. The classic (mercury) thermometer for air temperature measuring is mounted on the wall of monitoring centre [1], with resolution and accuracy of 0.5°C. Temperature data in this paper is obtained from this instrument.

Three normal plummets are installed on the dam, with labels NV1, NV2 and NV3 (Figures 4 and 5). They are located in sections 4, 5 and 11 [1]. Plummets are mechanically equipped with arc coordinate meters [2], designed by 'Telemac' [3], with resolution of 0.1 mm and measuring accuracy of 0.5 mm. All three plummets are in working conditions.

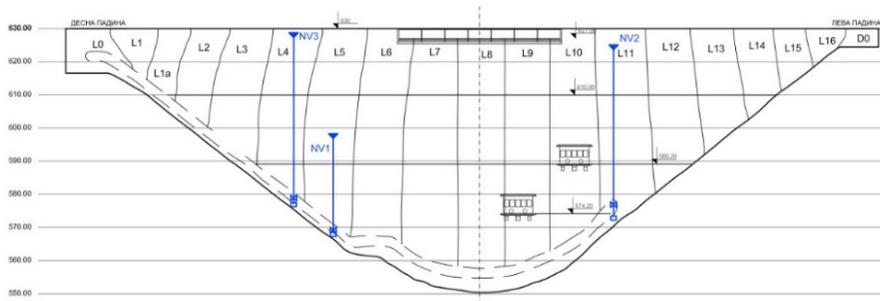


Figure 4. Plummet positions in developed dam plan

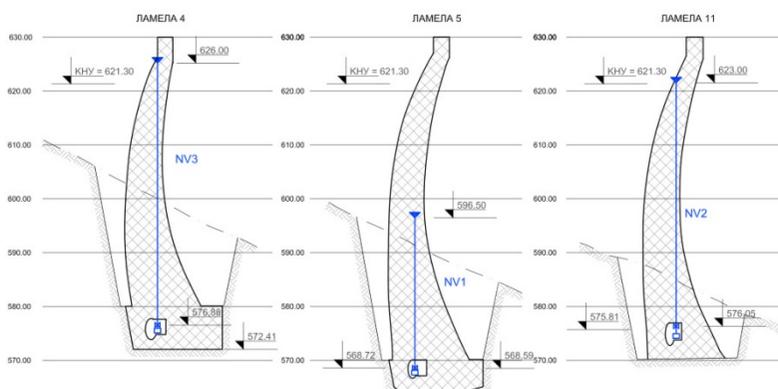


Figure 5. Plummet positions in sections 4, 5 and 11

#### 4. BEHAVIOUR OF DESCRIBED TECHNICAL MONITORING SYSTEM PARTS

The water level in the reservoir (Figure 6) changes within one year for about ten meters. According to the measurement data, the water level is mainly between elevations 608.0 m and 622.0 m. The highest recorded annual oscillation of the water level was about 20.0 m. The dam is located in the climate with a rather pronounced, not only seasonal, but also daily oscillations of air temperature (Figure 7). According to the measured data, the mean daily air temperature is in the range of  $-5^{\circ}\text{C}$  to  $+33^{\circ}\text{C}$ .

Relative horizontal displacements (Figure 8) measured by a plummet are within the limits of the values measured in the perennial period of dam monitoring. Registered changes in the period between two control measurements are in agreement with previously registered trends of changes in function of basic influential factors (seasonal and daily changes in temperature and reservoir water level). The largest radial displacement in the period between the two last control measurements was recorded on the plummet at NV2 and it had a value of 12.8 mm.

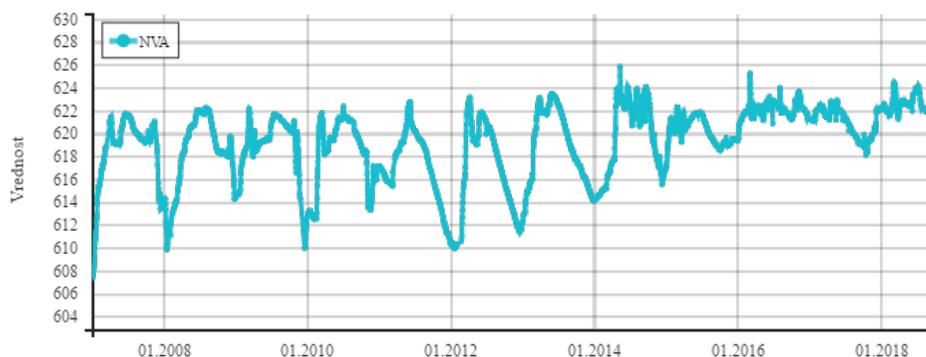


Figure 6. Reservoir water levels [mm], since 01.01.2007.

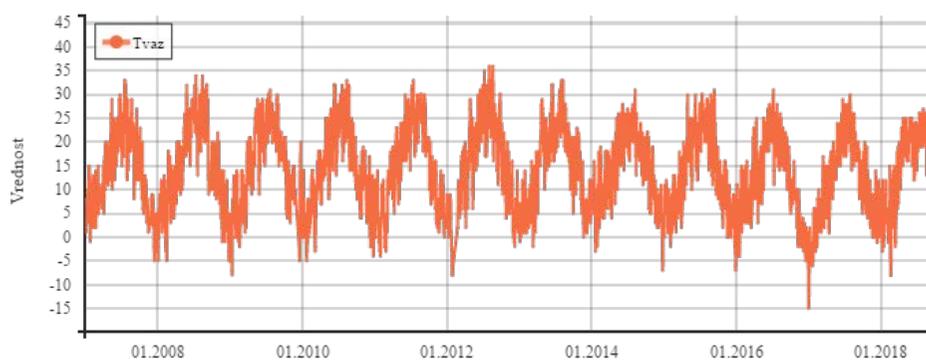


Figure 7. Air temperature [°C], since 01.01.2007.

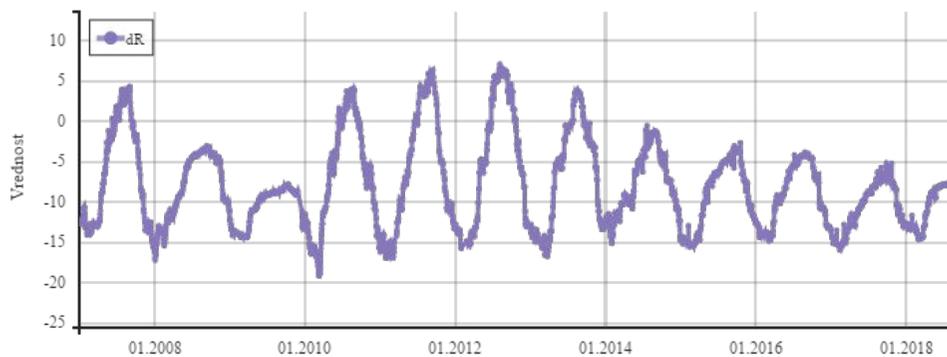


Figure 8. Radial displacement of plummet at NV2 [mm], since 01.01.2007.

## 5. STATISTICAL PREDICTIVE MODEL FOR RELATIVE HORIZONTAL DISPLACEMENTS

In this part of the paper statistical analysis of measurement results is described. Statistical models represent functional relationships obtained by methods of statistical analysis, between some causative variables and the observed values as consequences. Such models can be developed for diverse types of phenomena for which there is a sufficient data set. There are many papers dealing with this way of analysing behaviour of structures [4], [5], [6]. The main features of statistical models that make them suitable for dam monitoring are: they are relatively easy to use, they do not require complex computational processes and they can be applied to all variables. In order to develop a model, it is necessary to have a sufficient set of chronologically recorded data of causative and consequential variables. The basic causative variables that are taken into account in these models are: reservoir water level, monitoring time interval and air temperature.

Linear regression is a statistical approach to modelling the relationship between the scalar dependent variable  $Y$  and one or more independent variables  $X$ . In linear regression, data is modelled using predictor functions, and unknown parameters of the model are estimated based on known data.

In this paper, the radial displacement of plummet NV2 in the period from 01.01.2007. until 28.08.2018. is adopted as dependent variable, and reservoir water level and air temperature are adopted as independent variables. For statistical analysis of data, the "Expert Tool" software [7] developed by Water Institute "Jaroslav Černi" was employed. Based on the formed series of measured data (reservoir water level, air temperature) for the observed monitoring period, the predictor functions for causative variables are selected and coefficients for those functions are produced by statistical processing. In this way, the expected displacement-time function is defined, and it depends from the specified variables. Three predictive models have been developed: the first model takes into account only period of the year and number of days from the reference date; in the second model, the influence of reservoir water level was added; and in the third, the influence of air temperature on radial displacements was added. The predictor functions for each causative variable are given in Table 1:

Constant	$A_0$
Period of the year - drift ( $t$ )	$A_1 \cdot t, A_2 \cdot e^{-t/8}$
Period of the year - season ( $s$ )	$A_3 \cdot \sin(s), A_4 \cdot \cos(s)$
Reservoir water level ( $RWL$ )	$A_5 \cdot \text{avg}(RWL/360)$
Air temperature ( $T_{air}$ )	$A_6 \cdot T_{air} (-7), A_7 \cdot T_{air}^5$

Table 1. Predictor functions

where:

- $A_0 - A_7$  - coefficients of predictor functions determined by linear regression analysis,
- $t$  - number of days from reference date,
- $s = 2\pi d/365$  - period of the year between 0 and  $2\pi$ , from January 1<sup>st</sup> to December 31<sup>st</sup>,

- $d$  - number of days since January 1<sup>st</sup>,
- $\text{avg}(RWL/360)$  - mean reservoir water level in the last year,
- $T_{air}$  - air temperature,
- $T_{air}(-7)$  - air temperature delayed for 7 days relative to the measuring date.

Coefficient values for predictor functions of Model 1 are given in Table 2:

	Coefficient values	t-statistics	SD	F-statistics
<b>A<sub>0</sub></b>	17.722	12.590**	1.408	
<b>A<sub>1</sub></b>	-1.380	-19.543**	$7.059 \cdot 10^{-2}$	
<b>A<sub>2</sub></b>	-37.453	-16.116**	2.324	3105.20**
<b>A<sub>3</sub></b>	-4.830	-77.632**	$6.222 \cdot 10^{-2}$	
<b>A<sub>4</sub></b>	-4.613	-75.185**	$6.135 \cdot 10^{-2}$	

\*\* p<0.01

Table 2. Coefficient values for predictor functions and statistical indicators for Model 1

Coefficient values for predictor functions of Model 2 are given in Table 3:

	Coefficient values	t-statistics	SD	F-statistics
<b>A<sub>0</sub></b>	638.26	20.907**	30.528	
<b>A<sub>1</sub></b>	$-5.339 \cdot 10^{-1}$	-4.241**	$1.259 \cdot 10^{-1}$	
<b>A<sub>2</sub></b>	-24.673	-6.200**	3.979	2905.20**
<b>A<sub>3</sub></b>	-4.802	-83.284**	$5.766 \cdot 10^{-2}$	
<b>A<sub>4</sub></b>	-4.326	-75.797**	$5.708 \cdot 10^{-2}$	
<b>A<sub>5</sub></b>	-1.024	-19.598**	$5.225 \cdot 10^{-2}$	

\*\* p<0.01

Table 3. Coefficient values for predictor functions and statistical indicators for Model 2

Coefficient values for predictor functions of Model 3 are given in Table 4:

	Coefficient values	t-statistics	SD	F-statistics
<b>A<sub>0</sub></b>	578.93	19.321**	29.964	
<b>A<sub>1</sub></b>	$-5.381 \cdot 10^{-1}$	-4.396**	$1.224 \cdot 10^{-1}$	
<b>A<sub>2</sub></b>	-25.066	-6.479**	3.869	
<b>A<sub>3</sub></b>	-4.320	-65.620**	$6.583 \cdot 10^{-2}$	
<b>A<sub>4</sub></b>	-3.079	-29.573**	$1.041 \cdot 10^{-1}$	2225.10**
<b>A<sub>5</sub></b>	$-9.306 \cdot 10^{-1}$	-18.170**	$5.121 \cdot 10^{-2}$	
<b>A<sub>6</sub></b>	$9.827 \cdot 10^{-2}$	11.696**	$8.401 \cdot 10^{-3}$	
<b>A<sub>7</sub></b>	$6.276 \cdot 10^{-8}$	8.055**	$7.791 \cdot 10^{-9}$	

\*\* p<0.01

Table 4. Coefficient values for predictor functions and statistical indicators for Model 3

From presented tables, it can be noted that all important statistical indicators (F-statistics, t-statistics) are significant, which shows that all three models are statistically important, and that selected independent variables highly influence the dependent variable.

The coefficient of determination shows the variability percentage of the dependent variable that is explained by the model. As this value gets closer to 1, the model becomes more accurate. The value of the coefficient of determination in Model 3 is 0.813, which

shows that the obtained model is of good quality (and more accurate in relation to Models 1 and 2 for which obtained values are 0.762 and 0.802, respectively). In the developed form, Model 3 is presented as:

$$dR = 578.93 - 5.381 \cdot 10^{-1} \cdot t - 25.066 \cdot e^{-t/8} - 4.320 \cdot \sin(s) - 3.079 \cdot \cos(s) - 9.306 \cdot 10^{-1} \cdot \text{avg}(RWL|360) + 9.827 \cdot 10^{-2} \cdot T_{air}(-7) + 6.276 \cdot 10^{-8} \cdot T_{air}^5 [\text{mm}]$$

In Figure 9, the graphs (values shown in [mm] in function of time) of measured radial displacements of plummet at NV2 in the period from 01.01.2007 until 28.08.2018 (dR) are shown, as well as developed regression Model 3 (dR-regression) obtained from the predictor functions shown in Table 1 and corresponding coefficients for predictor functions shown in Table 4. In the diagram, around the dR-regression model, confidence interval is shown beyond which the measured data should not be found.

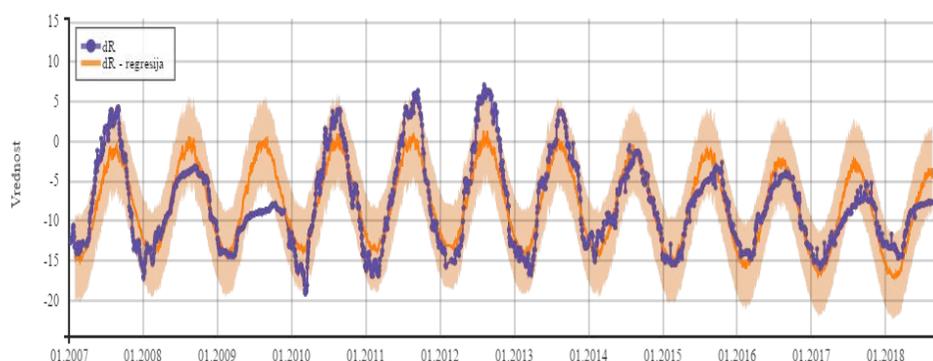


Figure 9. Series of measured values and values obtained by statistical Model 3 (with confidence interval) for radial displacements of plummet at NV2 in the period 01.01.2007. - 28.08.2018.

From the previous diagram, it can be observed that the measured values in 2009 deviated the most from the proposed statistical model. The reason for this behaviour of the structure is unknown but can certainly be analysed through some actions on the structure such as, for example, foundation settlement or previous local or regional seismic event.

Structure behaviour control in the case of statistical modelling approach is based on the comparison of measured and calculated values for each variable covered by the model.

If in some places there are no measured values, but independent variables are known, it is possible to determine the expected dependent variable (in this case, radial displacements) using the previously defined statistical model. The model can also be applied if we want to predict displacement values based on the assumed values of water level and air temperature.

## 6. CONCLUSION

In structural engineering, monitoring is performed to track changes that may occur during exploitation of a structure and in this way safety and reliability of a structure can be managed. By using the measurement results and through their processing, unwanted situations that could occur can be predicted. In this paper an overview of air temperature and reservoir water level influence on radial displacements of the dam is given. Measurement data were analysed for a monitoring period of about 12 years, but for a more detailed analysis, larger data series can be used, where greater clarity of the measured values over time will be obtained and the corresponding tendencies in the structural behaviour will be determined.

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## СТАТИСТИЧКИ ПРЕДИКЦИОНИ МОДЕЛ ХОРИЗОНТАЛНИХ ПОМЕРАЊА БРАНЕ „ВРУТЦИ“

*Резиме:* У овом раду је приказано формирање статистичког предикционог модела релативних хоризонталних померања бране „Врутци“ у функцији температуре ваздуха и нивоа воде у акумулацији као реверзибилних утицаја и од времена протеклог од почетка експлоатације бране, са којима су повезани иреверзибилни утицаји. Описани су поједини делови система техничког осматрања који су битни за израду поменутог модела са посебним освртом на њихово понашање. Представљена су мерења хоризонталних радијалних померања бране, температуре ваздуха и нивоа воде у акумулацији која чине серије података погодне за примену регресионе анализе. За статистичку анализу и одређивање коефицијента у предикционим функцијама коришћен је софтвер „Експертски алат“ развијен од стране Института за водопривреду „Јарослав Черни“. Формирана је серија са очекиваним вредностима мерења и утврђен степен њене усклађености са измереним подацима. Дискутован је утицај улазних величина на формирање предикционог модела.

*Кључне речи:* померања, ниво, температура, регресија, модел.