

## APPLICATION OF SMOOTHING TECHNIQUES ON REFERENCE EVAPOTRANSPIRATION IN SERBIA

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*Summary:* Irrigation systems represent a direct measure which can solve the needs for quality water in agriculture. In order to successfully design an irrigation system, it is necessary to analyze the current plant needs for water, i.e. evapotranspiration, as well as the future needs.

*In this paper, the FAO-56 Penman-Monteith method was used to determine the reference evapotranspiration ( $ET_0$ ) from 9 meteorological stations in Serbia for the period 1980-2010. The obtained  $ET_0$  values were used for  $ET_0$  analysis and forecasting. Three smoothing techniques were used (moving average method, exponential smoothing and Holt-Winters forecasting model), and three statistical tests were applied for their comparison.*

**Keywords:** reference evapotranspiration, FAO-56 Penman-Monteith, smoothing technique.

### 1. INTRODUCTION

The estimation of monthly and annual precipitation, evapotranspiration (ET) and rainfall deficit is essential for the irrigation system design. According to [1], ET is the prime component in a field of water balance and needs to be accurately quantified, especially in the context of irrigation management. ET is a process which depends on factors such as air temperature, relative humidity, solar radiation, wind speed and geographical position of place [2].

Plenty of authors have used different smoothing techniques for forecasting time series [1, 3-6]. In order to determine the most suitable models to generate forecast Murat et al. (2016) compared several exponential smoothing models on the data of air temperature, precipitation and wind speed from Jokioinen (Finland), Dikopshof (Germany), Lleida (Spain) and Lublin (Poland) [3]. They concluded that all analyzed methods are good for short time forecasting. Also, they concluded that best fitting model for precipitation

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depends on the site and that seasonal exponential smoothing model with no-trend should be used for prediction of air temperature and wind speed. Mohan and Arumugam (1995) [1] investigated the time variability of a weekly reference evapotranspiration ( $ET_0$ ) series in sub-humid location in south India and the applicability of a seasonal ARIMA model and a simple exponential smoothing model. Their results show that both models have satisfactory forecasting of  $ET_0$ , and the errors resulting from these forecasts were small, so the authors concluded that these models would be of much use in practice.

The main aim of this paper is to determine which smoothing technique is the best for use, analysis and forecasting on the  $ET_0$  time series. To define this, a comparative analysis of three smoothing techniques was implemented using the three statistical tests on the obtained results from nine meteorological stations in Serbia for the period from 1980 to 2010.

## 2. STUDY AREA

The climate in Serbia is moderately continental, with cold and short winters and hot summers. The coldest month is January with the mean monthly temperature of  $-6\text{ }^\circ\text{C}$  in the mountainous region, and  $0\text{ }^\circ\text{C}$  in the flat country regions. The warmest month is July with the mean monthly temperature ranging from 11 to  $32\text{ }^\circ\text{C}$ . The months with the maximum average sunlight are May, June, July, August and September. February and October are months with the lowest levels of humidity, while May and June have the highest levels of humidity.

Table 1 gives an overview of the geographical characteristics of nine meteorological stations through which the territory of Serbia was analyzed. The study period in this paper is from 1980 to 2010. All the data required for the calculation of  $ET_0$  values were taken from the meteorological yearbooks issued by the Republic Hydrometeorological Service of Serbia (RHMS).

*Table 1. Geographical description of the meteorological stations used in this study*

Station name	Longitude (E)	Latitude (N)	Elevation (m a.s.l.)
Palic	19.80	46.10	102
Vranje	21.92	42.55	432
Negotin	22.55	44.23	42
Nis	21.90	43.33	204
Dimitrovgrad	22.75	43.02	450
Beograd	20.47	44.80	132
Novi Sad	19.85	45.30	86
Loznica	19.23	44.55	121
Kragujevac	20.93	44.03	185

### 3. METHODOLOGY

#### 3.1. FAO-56 Penman-Monteith model

The  $ET_0$  values were estimated using the FAO-56 Penman – Monteith model [7]:

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T + 273} U_2 VPD}{\Delta + \gamma(1 + 0.34U_2)} \quad (1)$$

where  $ET_0$  – reference evapotranspiration ( $\text{mm day}^{-1}$ );  $\Delta$  – slope of the saturation vapour pressure function ( $\text{kPa } ^\circ\text{C}^{-1}$ );  $R_n$  – net radiation ( $\text{MJ m}^{-2}\text{day}^{-1}$ );  $G$  – soil heat flux density ( $\text{MJ m}^{-2}\text{day}^{-1}$ );  $\gamma$  – psychometric constant ( $\text{kPa } ^\circ\text{C}^{-1}$ );  $T$  – mean air temperature ( $^\circ\text{C}$ );  $U_2$  – average 24 h wind speed at 2 m height ( $\text{m s}^{-1}$ ) and VPD – vapour pressure deficit (kPa).

#### 3.2. Moving Average Method

The moving average (MA) method is based on the calculation of the average values from the observations [8]. In addition, the calculated average values are used as the predictors for the next period. In the MA method, each value has the same importance in the calculation. According to [9], the MA method can be calculated using the following equation:

$$F_{t+1} = (Y_t + Y_{t-1} + Y_{t-2} + \dots + Y_{t-n+1}) / n \quad (2)$$

where  $F_{t+1}$  - the forecast value for the next period,  $Y_t$  - the actual value at period  $t$ ,  $n$  - the number of terms in the moving average.

#### 3.3. Simple Exponential Smoothing Method

According to [10, 11], this method uses a weighted moving average of past data as a forecasting basis. The simple exponential smoothing (SES) method gives the greatest weight to more recent observations compared to observations from the more distant past [12]. The SES method can be calculated using the following equation:

$$F_{t+1} = \alpha Y_t + (1 - \alpha)F_{t-1} \quad (3)$$

where  $F_{t+1}$  - the new smoothed value or the forecast value,  $\alpha$  - the smoothing constant,  $Y_t$  - the new observation or actual value of the series in period  $t$ ,  $F_t$  - the old smoothed value or forecast for period  $t$ .

### 3.4. Holt-Winters Method

The Holt-Winters (HW) method is a smoothing technique which utilizes simple exponential smoothing to estimate the values of the level, trend and seasonality [13]. The Holt-Winters additive seasonal technique was used in this paper, and it was calculated using the following equations:

$$L_t = \alpha(Y_t - S_{t-s}) + (1 - \alpha)(L_{t-1} + b_{t-1}) \quad (4)$$

$$b_t = \beta(L_t - L_{t-1}) + (1 - \beta)b_{t-1} \quad (5)$$

$$S_t = \gamma(Y_t - L_t) + (1 - \gamma)S_{t-s} \quad (6)$$

$$F_{t+m} = L_t + b_t m + S_{t-s+m} \quad (7)$$

where  $L_t$  - smoothed estimate of the level at time  $t$ ,  $b_t$  - smoothed estimate of the change in the trend value at time  $t$ ,  $S_t$  - smoothed estimate of the appropriate seasonal component at  $t$ ,  $F_{t+m}$  - forecast for  $m$  period,  $\alpha$ ,  $\beta$ ,  $\gamma$  - smoothing constants,  $m$  - number of forecast,  $Y_t$  - the new observation or the actual value of the series in period  $t$ .

### 3.5. Measuring Forecasting Error

Three statistical tests were used for comparative analysis between the smoothing techniques (mean square error – MSE, mean absolute percentage error – MAPE, mean percentage error - MPE):

$$MSE = \frac{1}{n} \sum_{t=1}^n (Y_t - F_t)^2 \quad (8)$$

$$MAPE = \frac{1}{n} \sum_{t=1}^n \frac{|Y_t - F_t|}{Y_t} \quad (9)$$

$$MPE = \frac{1}{n} \sum_{t=1}^n \frac{(Y_t - F_t)}{Y_t} \quad (10)$$

where  $Y_t$  - the actual value in time period  $t$ ,  $F_t$  - the forecast value in time period  $t$ ,  $n$  - the number of forecast observations in the estimation period  $t$ .

#### 4. RESULTS AND DISCUSSION

The monthly  $ET_0$  values, at the daily level, for the period from 1980 – 2010, were treated as a time series and used for analysis. Figure 1 shows the monthly  $ET_0$  values for the Nis station. There are considerable variations of  $ET_0$  with regard to the time, and the seasonal effects are also noticeable.

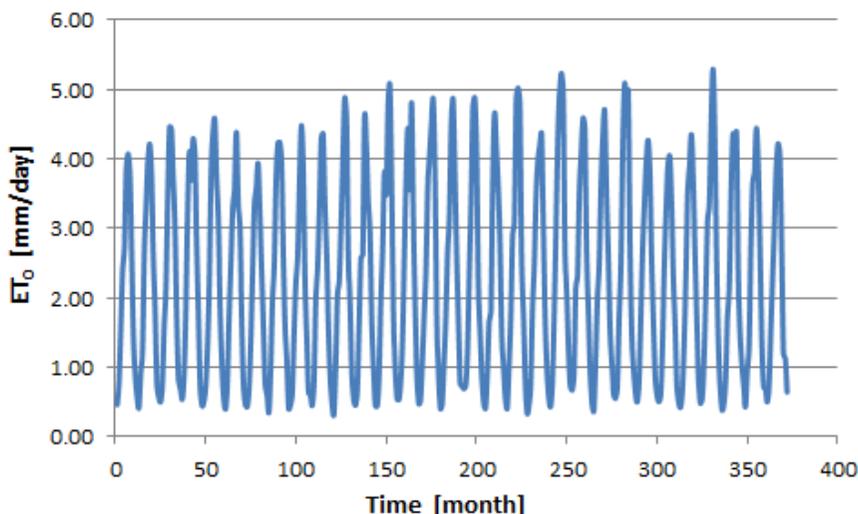


Figure 1. Monthly  $ET_0$  time series for the Nis station

Table 2 shows an overview of statistical parameters of the time series for all observed stations, through the four statistical tests. It can be noticed that the values of all the calculated statistical tests did not vary much for all stations. The highest mean value was observed at the Vranje station (2.55 mm/day), while the lowest was observed at the Loznica station. In addition, the highest values of standard deviation were observed at stations Palic and Vranje (1.63). According to table 2, the values of skewness are small and negative for kurtosis, which indicates that the distribution is right-tailed and has a more rounded peak and thinner tails.

Table 2. Statistics of time series for the observed meteorological stations

Station name	Mean	Standard deviation	Skewness	Kurtosis
Palic	2.43	1.63	0.27	-1.30
Vranje	2.55	1.63	0.37	-1.10
Negotin	2.29	1.58	0.36	-1.26
Nis	2.28	1.46	0.29	-1.28
Dimitrovgrad	2.42	1.46	0.29	-1.25

Beograd	2.51	1.54	0.27	-1.23
Novi Sad	2.39	1.54	0.25	-1.31
Loznica	2.07	1.44	0.33	-1.29
Kragujevac	2.15	1.37	0.30	-1.27

Figure 2 shows the values of autocorrelation and partial autocorrelation functions calculated for the Nis station. The autocorrelation function shows the presence of significant seasonal dynamics, and the function has peaks at lags equal to 12 and its multiples. The partial autocorrelation function does not cut off after one peak, which indicates a strong seasonality.

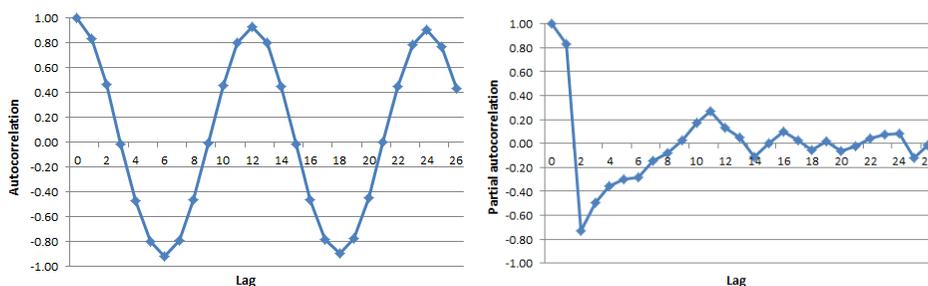


Figure 2. Autocorrelation and Partial autocorrelation of the time series for the Nis station

The original and smoothed (Holt-Winters seasonal additive)  $ET_0$  values for Nis are shown in figure 3. The smoothed values show a considerable deviation in the peaks with respect to time, especially during the period from 07/1990 to 07/2000. However, the results obtained using the Holt-Winters seasonal additive method show a closer correspondence between the smoothed and  $ET_0$  values.

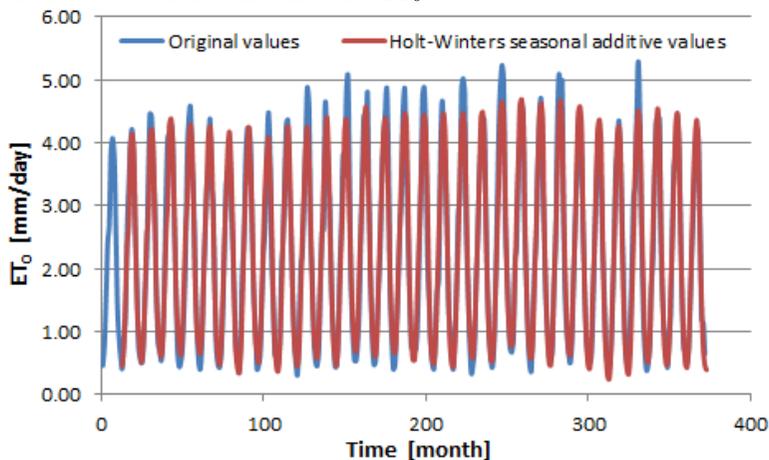


Figure 3. Original and Holt-Winters seasonal additive values of time series for the Nis station

The results of statistical tests for nine meteorological stations are shown in table 3. According to the statistical tests, the Holt-Winters seasonal additive method is the best smoothing method for the ET<sub>0</sub> time series in all the observed stations. The second place, after the HW, is occupied by the moving average, while the simple exponential method is in third place. In the HW method, the greatest similarity with the original time series, using the MSE test, is found at the Kragujevac station (0.08); at the Dimitrovgrad station (13.80) – using the MAPE test and at the Vranje station (-3.08) – using the MPE statistical test.

Table 3. Results of statistical tests for the observed smoothing techniques

Station	Statistics	Moving average (MA)	Simple Exponential (SES)	Holt-Winters (HW)
Palic	MSE	2.41	2.85	<b>0.13</b>
	MAPE	132.84	138.99	<b>19.00</b>
	MPE	-100.72	-101.13	<b>-7.08</b>
Vranje	MSE	2.44	2.87	<b>0.15</b>
	MAPE	103.83	110.69	<b>16.47</b>
	MPE	-73.32	-74.98	<b>-3.08</b>
Negotin	MSE	2.30	2.71	<b>0.12</b>
	MAPE	132.74	139.43	<b>19.61</b>
	MPE	-100.29	-101.41	<b>-4.70</b>
Nis	MSE	1.95	2.30	<b>0.10</b>
	MAPE	109.18	115.37	<b>14.46</b>
	MPE	-77.95	-78.91	<b>-3.94</b>
Dimitrovgrad	MSE	1.96	2.31	<b>0.13</b>
	MAPE	92.62	98.15	<b>13.80</b>
	MPE	-62.50	-63.07	<b>-3.52</b>
Beograd	MSE	2.16	2.55	<b>0.18</b>
	MAPE	100.69	105.89	<b>17.84</b>
	MPE	-70.29	-69.98	<b>-4.70</b>
Novi Sad	MSE	2.17	2.56	<b>0.16</b>
	MAPE	117.57	123.49	<b>19.05</b>
	MPE	-86.20	-86.50	<b>-4.58</b>
Loznica	MSE	1.89	2.24	<b>0.09</b>
	MAPE	137.77	144.61	<b>17.15</b>
	MPE	-105.22	-106.44	<b>-5.17</b>
Kragujevac	MSE	1.73	2.04	<b>0.08</b>
	MAPE	109.56	114.89	<b>14.28</b>
	MPE	-78.31	-78.35	<b>-3.89</b>

## 5. CONCLUSION

This paper presents the comparative analysis of three smoothing techniques (the moving average, the simple exponential smoothing and the Holt-Winters method) on the monthly  $ET_0$  values calculated at the daily level from nine meteorological stations in Serbia. The results of the statistical tests show that the Holt-Winters seasonal additive smoothing method is the best method, i.e. the HW has the best fitting of the smoothed and original time series. Unlike the HW method, the simple exponential method is the worst method (has the worst fitting between the smoothed and the original series) for all observed stations. Further studies will be oriented towards the analysis of robust techniques and artificial neural networks.

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## PRIMENA TEHNIKA UGLAČAVANJA KOD REFERENTNE EVAPOTRANSPIRACIJE U SRBIJI

**Rezime:** *Sistemi za navodnjavanje predstavljaju direktnu meru kojom je moguće rešiti potrebe za kvalitetnom vodom u poljoprivredi. Kako bi se uspešno isprojektovao sistem za navodnjavanje, neophodno je analizirati trenutne potrebe za vodom biljaka, tj. evapotranspiraciju, a takođe i buduće potrebe.*

*U ovom radu, za potrebe proračuna referentne evapotranspiracije ( $ET_0$ ) primenjena je metoda FAO-56 Penman-Monteith na podacima sa 9 meteoroloških stanica u Srbiji za period od 1980. do 2010. godine. Dobijene vrednosti  $ET_0$  iskorišćene su za sagledavanje  $ET_0$  i za predviđanje budućih vrednosti. Primenjene su tri tehnike uglačavanja (moving average method, exponential smoothing i Holt-Winters forecasting model), i tri statistička testa za njihovo poređenje.*

**Ključne reči:** *referentna evapotranspiracija, FAO-56 Penman-Monteith, tehnika uglačavanja.*