DESIGN METHODS FOR CROSS LAMINATED TIMBER ELEMENTS USED AS BEAMS

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UDK: 624.011.1.042  
DOI:10.14415/konferencijaGFS2017.020

Summary: In this paper load capacity of cross laminated timber beam element was investigated. Normal bending stress was calculated using: Simplified design method for calculating bending strength and Composite theory – k method. Three-layer cross laminated timber panel with height of 1 m and thickness of 94 mm was analyzed. The outer layers were 30 mm thick with identical mechanical properties. They were set in the longitudinal direction, perpendicular to the direction of the bending force. Inner layer had thickness of 34 mm and with longitudinal layers form a right angle.

Keywords: cross laminated timber, beam element, design methods

1. INTRODUCTION

Wood is one of the oldest building materials and along with stone it was the basic building material for many years. Its characteristics allow for a high degree of prefabrication, quick assembly, and immediate utilization. Wood has great fire resistance, and during the fire retains its characteristics, i.e. its mechanical properties do not change significantly due to high temperatures. Timber constructions are five times lighter than reinforced concrete, thus they are better capable of weathering seismic forces and stand out as material of choice for earthquake prone areas. Timber constructions have high energy efficiency. Within last few decades, wood has been increasingly used in modern architectural buildings (sports’ arenas, residential buildings and bridges) thanks to better understanding of wood as a material, utilization of modern timber construction and high quality connections. Construction elements for contemporary timber constructions are primarily based upon contemporary products such as glued laminated timber and cross laminated timber.

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2. CROSS LAMINATED TIMBER AS A BUILDING MATERIAL

Cross laminated timber (CLT) is a sophisticated modern product that greatly improves the physical properties of traditional wood as building material. CLT is made of controlled dried wooden elements – layers of uniform width, free of defects (knots, resin etc.). By removing all of the defects and cross-gluing the layers it is possible to produce the material that has more uniform mechanical properties than the traditional wood.

A CLT cross section contains a minimum of three cross-glued layers, but most often five or seven. Alternate layers are mutually perpendicular; however, it is possible to double certain layers so that greater strength can be achieved in a desired direction, Figure 1. Laminate height within a CLT varies from 16 to 51 mm while width varies between 60 mm and 240 mm.

![Figure 1. CLT panel cross sections [1]](image)

The primary direction of the load-bearing capacity generally corresponds to the outer layer orientation. To meet aesthetic, fireproof, and insulation requirements, the exposed surfaces of the panels can be covered with gypsum or similar appropriate finish.

CLT is most often manufactured from conifers grade C24, and moisture content 12±2%, [2]. Panel dimensions are dictated by manufacturer technology and mode of transportation. Worldwide, CLT became greatly popular for the following reasons: CLT is much stronger with far more superior mechanical properties than traditional wood, no bending tendency, minimal fractures appearance, high resistance to fire, and resilience to earthquakes.

Because of the increasing number of different types of CLT panels a general design method is needed for practice. In the following, two design methods for CLT elements used as beams are presented.
3. ANALYTICAL DESIGN METHODS FOR CLT BEAMS

CLT elements under axial in-plane loads acting as deep beams can be designed using different approaches. The following two methods are most frequently used for CLT beams design, [3] [4]: Simplified design method for calculating bending strength and Composite theory – k method. Figure 2 illustrates two 3-layer CLT systems under in-plane bending loads. The same configurations are possible for 5- and 7-layer CLT panels.

![Figure 2. CLT panels (beams) under axial in-plane loads [1]](image)

3.1 Simplified design method for calculating bending strength

The bending stress may be expressed as:

\[
\sigma = M \cdot y \cdot \frac{E_{\text{mean}}}{(EI)_{\text{eff}}}
\]

(1)

where \( E_{\text{mean}} \) is the mean modulus of elasticity of the longitudinal layer in tension and \( (EI)_{\text{eff}} \) is determined using the net cross-section.

The maximum stress will occur for \( y = H / 2 \) where \( H \) is the beam depth, therefore equation (1) becomes

\[
\sigma_{\text{max}} = M \cdot 0.5H \cdot \frac{E_{\text{mean}}}{(EI)_{\text{eff}}}
\]

(2)
When the modulus of elasticity of all longitudinal layers are equal, then equation (2) can be expressed as:

\[
\sigma_{\text{max}} = M \cdot 0.5H \cdot \frac{1}{I_{\text{eff}}}
\]  

(3)

and \( I_{\text{eff}} \) can be calculated as:

\[
I_{\text{eff}} = \frac{h_{\text{eff}} \cdot H^3}{12} = \frac{H^3}{12} \sum_i h_i
\]  

(4)

where \( H \) is the beam depth and \( h_i \) is the thickness of boards perpendicular to the axial load (effective boards), Figure 3.

![Effective boards in CLT panel under axial in-plane load](image)

Figure 3. Effective boards in CLT panel under axial in-plane load [4]

This method assumes a composite action between effective longitudinal boards. More conservative way to evaluate the \( I_{\text{eff}} \) would be to sum the individual moments of inertia of all effective boards.

### 3.2 Composite theory – k method

The values for stress and stiffness can be calculated by using the coefficient \( k_i \), Table 1.

<table>
<thead>
<tr>
<th>Load in the plane of a panel</th>
<th>Grain direction</th>
<th>Effective Normal Stress</th>
<th>Effective Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending</td>
<td>Along the grain</td>
<td>( \sigma_{m,0,\text{ef}} = \sigma_{m,0} \cdot k_3 )</td>
<td>( E_{m,0,\text{ef}} = E_0 \cdot k_3 )</td>
</tr>
<tr>
<td></td>
<td>Across the grain</td>
<td>( \sigma_{m,90,\text{ef}} = \sigma_{m,0} \cdot k_4 )</td>
<td>( E_{m,90,\text{ef}} = E_0 \cdot k_4 )</td>
</tr>
</tbody>
</table>
Depending on the position of the outer layer, coefficients $k_i$ are defined in Table 2.

**Table 2. Coefficients $k_i$ Values**

<table>
<thead>
<tr>
<th>Load</th>
<th>$k_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td>[ k_3 = 1 - \left(1 - \frac{E_{90}}{E_0}\right) \frac{a_{m-2} - a_{m-4} + \ldots \pm a_1}{a_m} ]</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td>[ k_4 = \frac{E_{90}}{E_0} + \left(1 - \frac{E_{90}}{E_0}\right) \frac{a_{m-2} - a_{m-4} + \ldots \pm a_1}{a_m} ]</td>
</tr>
</tbody>
</table>

This design method includes the stiffness of all layers. Modulus of elasticity of lateral layers can be calculated as:

\[ E_{90} = \frac{E_0}{30} \]  

Figure 4 depicts the method to determine spans $a$ of CLT panel with 5 layers ($m = 5$).

The maximum effective bending stress may be expressed as:

\[ \sigma_{\text{max,eff}} = \frac{M}{W} \]  

where $W$ can be calculated as:

\[ W = \frac{a_m \cdot H^2}{6} \]  

where $a_m$ is the total thickness of the CLT panel and $H$ is the beam depth.
4. NUMERICAL EXAMPLE

Calculate in-plane bending stress of a three-layer CLT beam panel with 94 mm (30+34+30) thickness and 1000 mm depth, Figure 3. Bending moment is 100 kNm.

Physical and mechanical properties

Longitudinal layers:

\[ E_0 = 11000 \text{ MPa} \]
\[ E_{90} \approx \frac{E_0}{30} = \frac{11000}{30} = 367 \approx 370 \text{ MPa} \]
\[ G_0 \approx \frac{E_0}{16} = \frac{11000}{16} = 688 \approx 690 \text{ MPa} \]
\[ G_R \approx \frac{G_0}{10} = \frac{690}{10} = 69 \text{ MPa} \]

Lateral layers:

\[ E_0 = 9000 \text{ MPa} \]
\[ E_{90} \approx \frac{E_0}{30} = \frac{9000}{30} = 300 \text{ MPa} \]
\[ G_0 \approx \frac{E_0}{16} = \frac{9000}{16} = 563 \approx 560 \text{ MPa} \]
\[ G_R \approx \frac{G_0}{10} = \frac{560}{10} = 56 \text{ MPa} \]

Simplified design method for calculating bending strength

The modulus of elasticity of all longitudinal layers is equal so \( I_{\text{eff}} \) can be calculated as:

\[ I_{\text{eff}} = \frac{h_{\text{eff}} \cdot H^3}{12} = \frac{H^3}{12} \sum h_i \]

where \( H \) is the beam depth equal to 100 cm and \( h_i \) is the thickness of boards perpendicular to the axial load:

\[ I_{\text{eff}} = \frac{100^3}{12} \cdot (3 + 3) = 5 \cdot 10^5 \text{ cm}^4 \]

In that case, maximum bending stress is:

\[ \sigma_{\text{max}} = M \cdot 0.5H \cdot \frac{1}{I_{\text{eff}}} = 10 \cdot 10^6 \cdot 0.5 \cdot 100 \cdot \frac{1}{5 \cdot 10^5} = 1000 \text{ N/cm}^2 \]
4.1 Composite theory – k method

The spans $a$ of CLT panel with $m = 3$ are:

$$a_m = a_3 = 94 \, mm$$
$$a_{m-2} = a_1 = 34 \, mm$$

For the longitudinal boards of CLT panel, coefficient $k_3$ is defined in Table 2:

$$k_3 = 1 - \left( 1 - \frac{E_0}{E_{90}} \right) \frac{a_{m-2} - a_{m-4} + \ldots \pm a_1}{a_m} = 1 - \left( 1 - \frac{300}{11000} \right) \frac{34}{94} = 0.6482$$

The maximum effective bending stress is:

$$\sigma_{\text{max,eff}} = \frac{M}{W}$$

where $W$ can be calculated as:

$$W = \frac{a_m \cdot H^2}{6} = \frac{9.4 \cdot 100^2}{6} = 15667 \, cm^2$$

In that case,

$$\sigma_{\text{max,eff}} = \frac{10^7}{15667} = 638.28 \, N/cm^2$$

and the maximum bending stress defined in the Table 1 is:

$$\sigma_{\text{max}} = \frac{\sigma_{\text{max,eff}}}{k_3} = \frac{638.28}{0.6482} = 984.7 \, N/cm^2$$

5. CONCLUSION

Use of CLT creates healthy and natural ambiance. Timber’s great advantage is that it is a renewable resource, and in the process of growth trees absorb great amount of carbon dioxide. Compared against traditional wood, CLT constructions are stronger, have better mechanical properties, no bending tendency, and minimal fractures appearance. This work presents two analytical design methods for CLT elements used as beams: Simplified design method for calculating bending strength and Composite theory – k method. Both methods are simple and give similar results. Future research will include the analysis of the dynamic load capacity of cross laminated timber beam element.
REFERENCES


METODE PRORAČUNA UNAKRSNO LAMELIRANIH GREDNIH ELEMENATA

Rezime: U ovom radu ispitana je nosivost unakrsno lameliranog drvenog grednog elementa. Proračun normalnog napona savijanja izvršen je pojednostavljenom metodom i K-metodom. Analiziran trolojni unakrsno lamelirani gredni element je visine 1 m a debljine 94 mm. Spoljne lamele su debljine 30 mm, identičnih fizičkomehaničkih karakteristika. Postavljene su u podužnom pravcu, normalno na pravac delovanja sile savijanja. Unutrasnja lamela je debljine 34 mm i sa podužnim lamelama obrazuje prav ugao.

Ključne reči: unakrsno lamelirano drvo, gredni elementi, dimenzionisanje