COMPARATIVE ANALYSIS OF THE ELLIPSOID APPROXIMATION WITH THE SPHERE

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ABSTRACT:

The paper analyzes the approximation of the ellipsoid by the sphere. Earth is a space body with a mathematically irregular shape, so idealized smooth surfaces are used for calculations. The first is the geoid, a smooth, equipotential surface that best approximates mean sea level. However, the geoid does not have an analytical form and is unsuitable for many applications, so an ellipsoid is used for approximation. In applications where high accuracy is not required (e.g., with small scale maps), the ellipsoid is approximated by a sphere. The radius of the sphere can be calculated in three ways: according to the equivalent volume criterion, according to the equivalent surface criterion, or as the mean value of the three semi-axes of the ellipsoid. All three methods of approximation were tested by calculating the length of the geodetic line on the ellipsoid, the orthodrome on the spheres and then the error. Also, the influence of latitude on the error value was tested. For three different values of geographic latitude, the lengths of geodetic lines up to one hundred points were calculated (using the Bessel method for solving the second main geodetic task on the ellipsoid), as well as the lengths of the orthodromes on the spheres, with the radii of the spheres determined in the three mentioned ways. The obtained results were then analyzed and discussed.

KEYWORDS:

orthodrome, loxodrome, geodetic line, approximation, analysis

1 INTRODUCTION

The equipotential surface that best approximates the mean sea level for the entire Earth's surface is called the geoid (Figure 1). Gauss defined the geoid as the mathematical shape of the Earth and as such it represents a key surface in geodesy, with a particularly important role in height positioning. In the first approximation, i.e., to the nearest few meters, the geoid represents mean sea level. In the general case, it passes under the continents at a depth equal to the height, i.e., at sea level and possesses all the properties of an equipotential surface, i.e., surface of constant scalar potential [1].

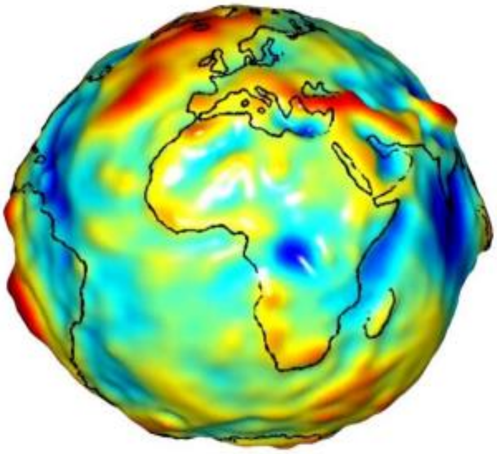

Figure 11. Geoid

Although geodetic measurements are performed on the physical surface of the Earth, that surface is unsuitable for mathematical processing and calculations due to its geomorphological complexity. Therefore, the calculations are performed on a regular mathematical surface after the measurements are reduced to it [1]. The choice of the shape of a regular mathematical surface is in principle arbitrary, but for practical reasons it is required to be mathematically as simple as possible, and to partly or fully approximate the real Earth [2]. The simplest mathematical body whose shape resembles the real Earth is a two-axis rotating ellipsoid. It is created by rotating the ellipse around its minor axis [3]. Moreover, if the ellipsoid is chosen to approximate the entire Earth, then it is called the general (global) Earth ellipsoid [1]. When it comes to a part of the Earth, such as the territory of a country or continent, the ellipsoid is called local (Figure 2).

In some cases, e.g., when studying cartographic projections and the construction of cartographic networks for small-scale maps, the flattening of the Earth's ellipsoid can be ignored and the Earth can be considered a ball of the appropriate radius. The radius of the Earth is determined in several ways [2]. The most commonly applied solutions are:

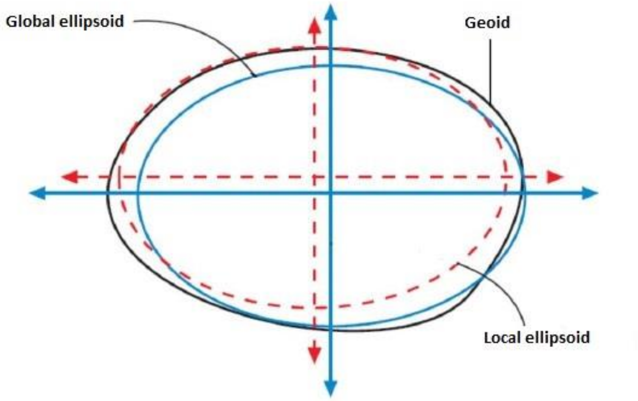

Figure 12. Geoid, local and global ellipsoid

a) The radius of the globe that has the same volume as the Earth's ellipsoid:

$$R_v = \sqrt[3]{a^2 b} \tag{1}$$

Where a is the length of the semi-major axis, and b is the length of the semi-minor axis.

This expression follows from the equations for the volume of the ellipsoid (V_e) and the ball (V_l):

$$V_e = \frac{4}{3}\pi a^2 b \tag{2}$$

$$V_l = \frac{4}{3}\pi R^3 \tag{3}$$

b) The radius of the globe from the equivalent surface of the ellipsoid:

$$4\pi R_p^2 = 4\pi a^2(1-e^2)(1+\frac{2}{3}e^2+\frac{3}{5}e^4+\frac{4}{7}e^6+\cdots) \tag{4}$$

Where *e* is the flatness coefficient and whence follows:

$$R_p = a\sqrt{(1-e^2)(1+\frac{2}{3}e^2+\frac{3}{5}e^4+\frac{4}{7}e^6+\cdots)} \tag{5}$$

c) The radius of the Earth's sphere can also be determined as the arithmetic mean of the three semi-axes of the revolving ellipsoid:

$$R_s = \frac{a+a+b}{3} \tag{6}$$

Also, when making geographical maps on a very small scale, even for a relatively large area of territory, the Earth can be approximated by a ball with a radius of *R* ≈ 6 370 km, that is, *R* ≈ 6 371 km [3].

A certain number of lines on the ellipsoid and their corresponding lines on the ball have special characteristics that are significant for study and analysis in this paper. These are orthodrome, loxodrome and geodetic line (Figure 4).

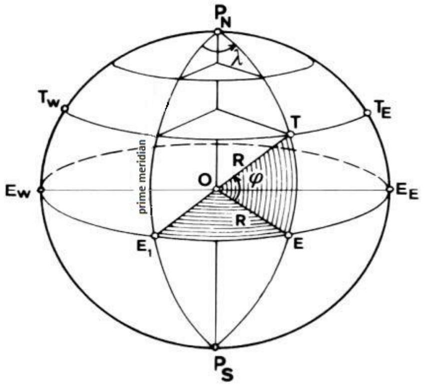

Figure 13. System of geographic coordinates on a ball (sphere)

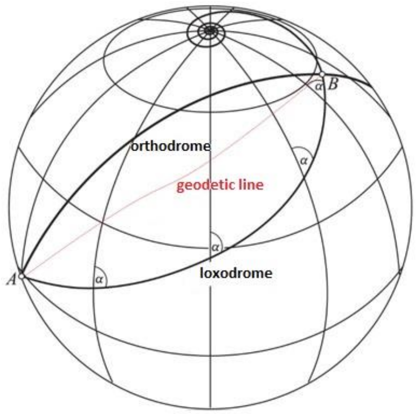

Figure 14. Orthodrome, loxodrome and geodetic line

A geodetic line represents a curve on a given surface, at each point of which the main normal of the curve coincides with the corresponding normal on the surface [3]. The main characteristic of a geodetic line is that it represents the shortest line connecting given points on any analytically determined surface, and this is its first property [4]. If it is a geodetic line located on the Earth's ellipsoid, its second characteristic is that for each of its points the product of the radius r of the parallel of the corresponding point and the sine of the azimuth of the geodetic line at the same point is constant [3]:

$$r \sin \alpha = N \cos \varphi \sin \alpha = const = C \tag{7}$$

Where *N* is the radius along the prime vertical.

This expression represents Clair's equation of the geodetic line. Due to its characteristic property, the geodetic line describes an ellipsoid (Figure 5), but due to the eccentricity of the ellipsoid, it does not repeat itself during that cycle [3].

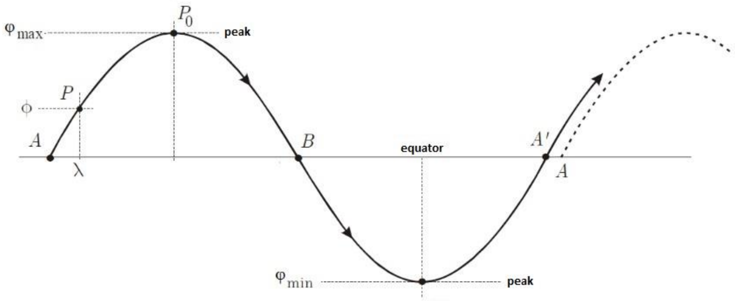

Figure 15. Geodetic line cycle

The curve on the surface of the Earth's ellipsoid, which intersects all meridians at the same angle (azimuth), is called a rhombus [5]. This feature makes it suitable for navigation, as it allows traveling (sailing or flying) on it with a constant course. However, it does not represent the shortest connection between two points, which means that the journey along the loxodrome takes longer and is therefore more expensive [6]. In some cartographic projections, the loxodrome is shown as a straight line, which enables its combined use with the orthodrome for navigational purposes, with the simultaneous application of the feature of the shortest path of the orthodrome and the constancy of the loxodrome course [7]. Due to multiple applications, mathematical models are considered, i.e., rhombus equations on the ellipsoid, ball and projection plane [8].

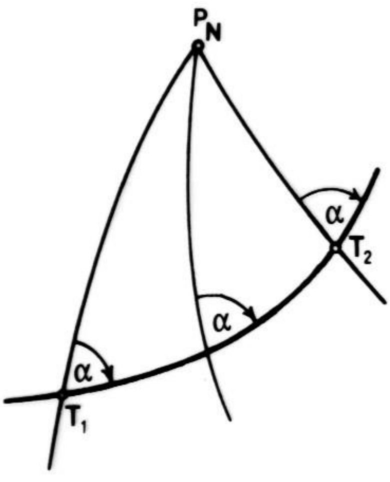

Figure 16. Loxodrome

The term orthodrome comes from the Greek words ortos-dromos and means a straight path. The shortest path between two points is always a part of a circle, on any ball, i.e., a shorter arc of the great circle for that ball [6].

2 ORTHODROME

The orthodrome is defined with the help of imaginary circles on the Earth, which are divided into small and large circles, that is, into a small and a large circle [6]:

a) Great circle - all circles on the surface of the Earth that have a common center at the center of the Earth. These are the meridians, the equator and the orthodromes (Figure 7).

b) Small circle - all circles whose center is in the Earth's axis. These are parallels.

In mathematical cartography, for the shortest distance between two points on the ball, i.e., for the arc of a great circle, the term orthodrome (great circle) is used. A great circle is a circle on the surface of a sphere, i.e., a ball, which divides the sphere into two equal hemispheres and has the same center as the sphere. In other words, a great circle is the intersection of a sphere with a plane passing through its center [5].

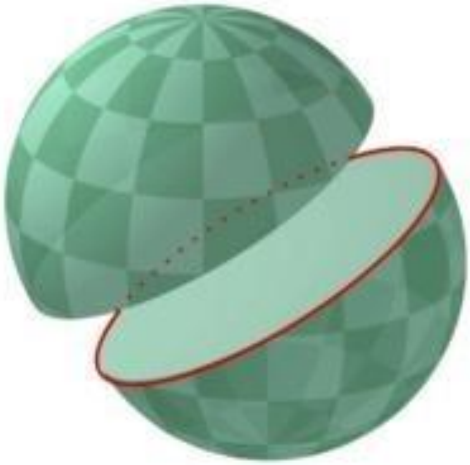

Figure 17. A great circle that divides the sphere on two equal parts

Orthodromes are easily identified on the globe based on lines of longitude and latitude. Each line of longitude, or meridian, is the same length and represents half of a great circle [7]. This is because each meridian has a corresponding line on the opposite side of the Earth, and when these two lines meet, they cut the ball into equal halves, making a great circle. The only line of latitude, or parallel, that is characterized as a great circle is the equator, because it passes right through the center of the Earth and divides it in half. The lines of latitude north and south of the equator are not great circles because their length decreases as they move toward the poles, and they do not pass through the center of the Earth either. As such, these parallels are considered small circles [8].

An orthodrome is a shorter arc of a great circle that passes through two points on Earth [6]. Only one orthodrome can pass through two points on Earth. An orthodrome is fully defined if the orthodrome length, initial azimuth, start and end points of the orthodrome are known (if they are not diametrically opposite, otherwise another point is required). Orthodromic length is expressed in degrees, kilometers or nautical miles [8]. As the shortest distance on any analytic surface between two points is defined as a geodetic line, the orthodrome is a geodetic line on the globe and is part of a great circle through those two points (Figure 8).

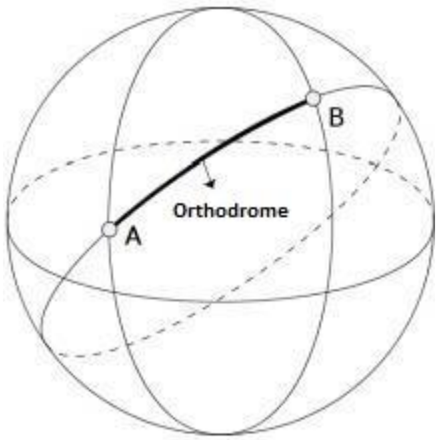

Figure 18. Orthodrome on Earth's ball

For example, in the Mercator projection, the orthodrome is shown as a rounded curve, bulging towards the pole (Figure 9), and on the map, whose projection center is in the center of the Earth, it is shown as a straight line [9]. Due to the lengthening of the Mercator map, it is not recommended to measure the distances between intermediate points directly, because they are large values [10]. The advantage of the map is that on it every big circle is shown as a straight line, and because of this, it is very easy to draw the orthodrome along which you want to travel. The disadvantage of the map is that you cannot read courses or measure distances on it [11].

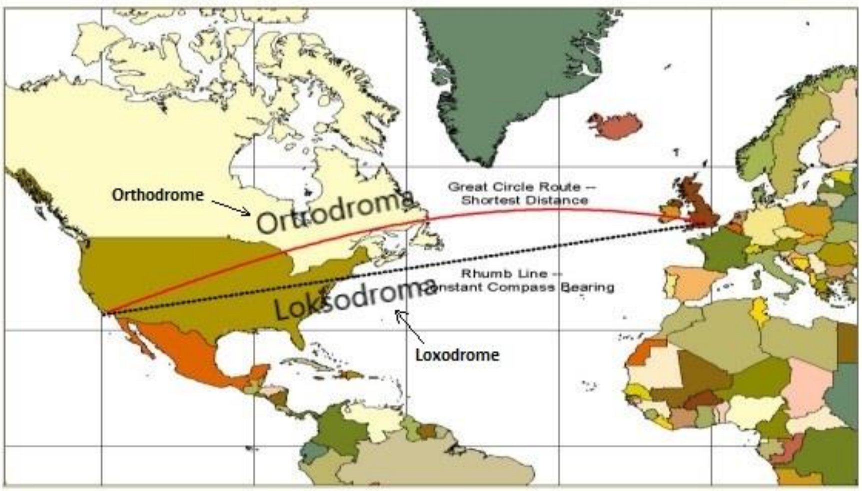

Figure 19. Orthodrome and loxodrome in Mercator projection

The length of the orthodrome is a very important factor when planning a travel route (e.g., sailing plan or flight plan). It is usually associated with savings depending on the loxodrome route. The starting point $A(\varphi_1, \lambda_1)$ and the end point $B(\varphi_2, \lambda_2)$ are required for calculation, after which the length of the orthodrome, i.e., the shortest distance between those two points is calculated according to one of the forms of spherical trigonometry, usually using the cosine expression:

$$D_o = R \cdot \cos^{-1} \alpha \cdot \sin \varphi_1 \cdot \sin \varphi_2 + \cos \varphi_1 \cdot \cos \varphi_2 \cdot \cos \Delta\lambda \quad (8)$$

The size D_o represents the required length of the great circle, i.e., the length of the orthodrome, in the unit of measurement of length, the size $\Delta\lambda$ is the difference in the longitude of the starting and ending points, and *R* is the radius of the Earth [5].

2.1 APPLICATION OF ORTHODROME

Orthodrome has been an important part of navigation and geography for hundreds of years, and knowledge about it is extensive, i.e., essential for long trips around the world. The first disadvantage of the orthodrome is that all meridians intersect at different angles, so theoretically one should change course continuously [2]. In practice, along with the orthodrome, the so-called intermediate points, and between them one sails along a loxodrome, so that one long orthodrome is divided into more or less shorter loxodromes [8]. Another disadvantage of the orthodrome is that it leads to high latitudes, often a more dangerous area of navigation. In the event that part of the orthodrome is at too wide a width and there is a risk of bad meteorological conditions, combined sailing will be chosen, i.e., combination of loxodrome and orthodrome. In the case of combined navigation, the border parallel over which one intends to sail is mostly determined, and from it and to it one sails along the orthodrome, and along it (between the border points) one sails along the loxodrome - a special case of parallel navigation [7].

Special cases of the orthodrome are sailing along the equator and meridian, and in those cases all calculations are done very simply. Traveling on the orthodrome is more difficult, because it is necessary to constantly change direction. An exception is traveling along the meridian or the equator, because they are also great circles, i.e., orthodromes. In those cases, orthodromes and loxodromes coincide. While ships were slow and world trade was not as intense as it is today, the advantage of the orthodrome as a shorter sea route was less pronounced [7]. However, with the increase in the carrying capacity of ships, the increase in fuel consumption and the need for faster cargo manipulation, any unnecessary detention of the ship became expensive, so the importance of the shortest sailing route also increased.

Orthodromes in the North Atlantic connect the ports of the East Coast of the USA and Western Europe. Due to the relative shortness, the savings are around a hundred nautical miles, however, due to entering higher latitudes, rhombus sailings often have their own justification. Orthodromes in the Middle and South Atlantic connect the ports of Southern Europe and West Africa with the ports of North, Central and South America. Due to the low latitudes, the savings are not particularly large and amount to up to a hundred nautical miles. Orthodromic savings in the North and South Pacific are particularly significant. On these routes, ships can save up to 500 nautical miles when sailing on the orthodrome. The biggest savings are on orthodromic routes that lead from ports in South America to ports in Australia, Indonesia, and New Zealand [9].

3 EXPERIMENTAL PART

As part of the experimental part of the work, scripts were created in the Matlab software package that calculated the lengths of the geodetic line on the ellipsoid and the orthodrome between the central and peripheral points (Figure 10). Three central points were chosen at approximately 30°, 45°, and 60° north latitudes. Although it was possible to simply specify the values of the coordinates of the central points, the cities at the desired latitudes were selected, and their coordinates were taken from the Google Earth software.

Since the radii of curvature of ellipsoids change with latitude, it is possible to obtain different results. Although longitude has no effect on the result, the values of this coordinate are very similar for all three central points. From each central point, the longitudes of up to 100 peripheral points were calculated, which were selected so that the first one (with an index of 1) is located on the same meridian as the central point, and the latitude is 5 degrees higher. Each subsequent peripheral point has a latitude less than 0,1°, and a longitude greater than 0,1°, until it is on the same parallel as the central one. Then, to the point with index 100, both width and length are reduced by 0,1°.

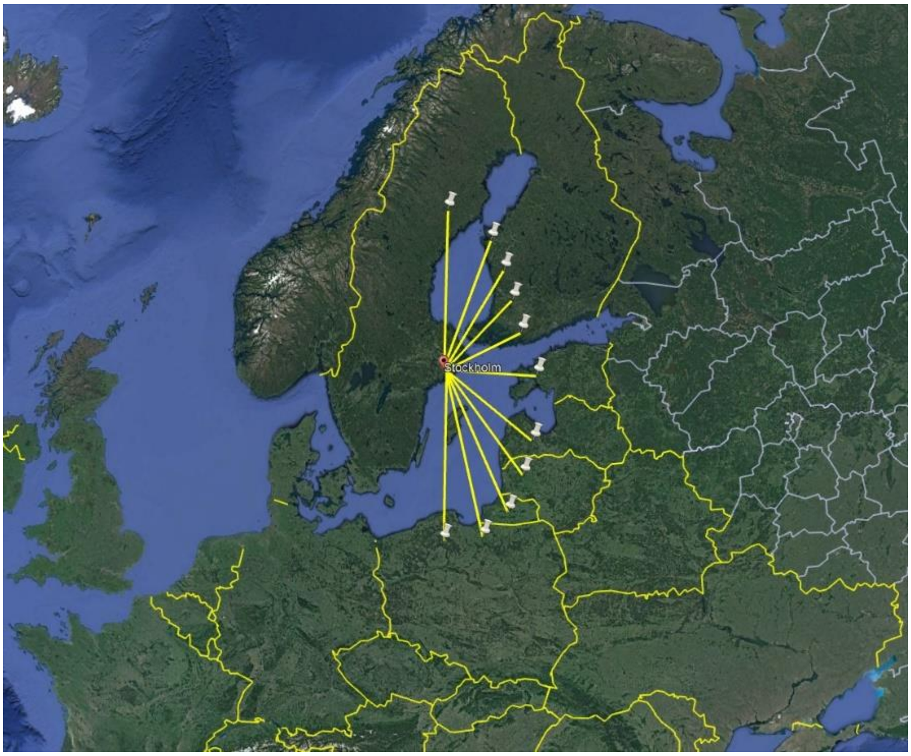

Figure 20. Central point CT1 (Stockholm)

The length of the geodetic line was calculated on the GRS80 ellipsoid using the Bessel method for solving the second main geodetic task. The radii of the spheres that approximate the GRS80 ellipsoid were calculated according to expressions (1) (radius R_1),

(5) (radius R_2) and (6) (radius R_3), and the length of the orthodrome according to expression (7). The differences between the lengths of the geodetic line and the orthodrome were then calculated and the obtained values were analyzed.

3.1 RESULTS

Due to the large number of obtained numerical values, they are shown graphically, and later in the discussion, the values that are of interest for the analysis are highlighted. The calculated radii of the spheres used in calculating the lengths of the orthodrome are R_1 = 6 371 000,790 m, R_2 = 6 371 000,181 m and R_3 = 6 371 008,771 m.

Stockholm was chosen as the first central point (CT1), at 59˚20'00''N and 18˚03'00''E. At this latitude, the radius of the bend along the meridian of the GRS80 ellipsoid is M = 6 351 187,187 m, and along the first vertical N = 6 393 990,993 m.

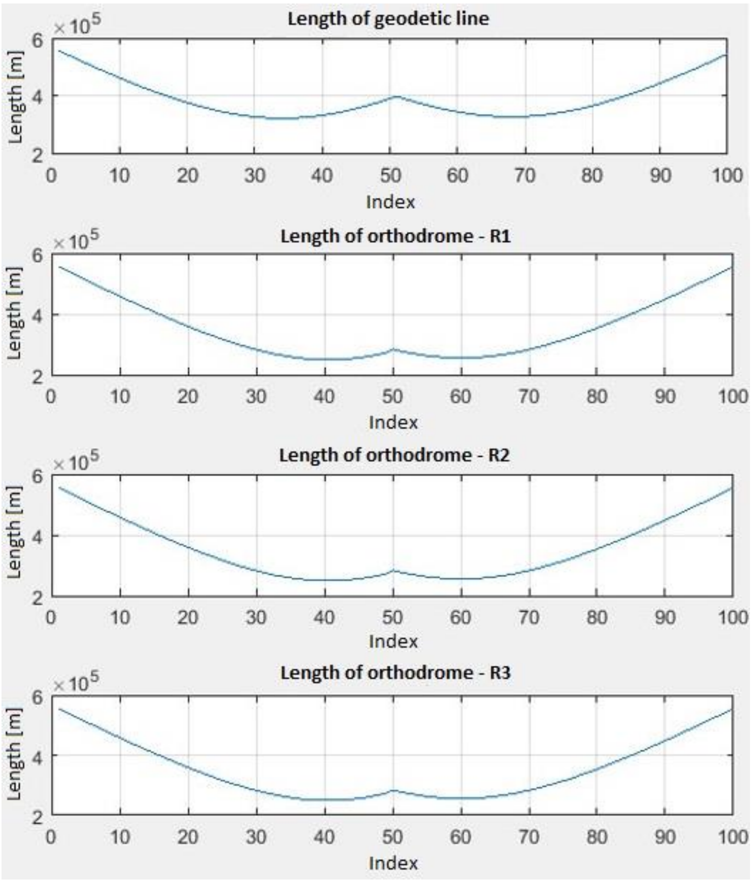

Figure 21. The lengths of geodetic lines and orthodromes for central point CT1

Novi Sad was selected as the second central point (CT2), at 45˚15'06''N and 19˚50'13''E. At this latitude, the radius of curvature along the meridian of the GRS80 ellipsoid is M = 6 346 162,595 m, while along the first vertical N = 6 388 932,537 m.

Table 3. Values of geodetic line lengths and orthodromes to individual peripheral points for CT1

	Point	Lengths of geodetic lines [m]	Lengths of orthodromes – R1 [m]	Lengths of orthodromes – R2 [m]	Lengths of orthodromes – R3 [m]
CT1 (Stockholm)	1	556 152,577	555 974,702	555 975,259	555 975,398
	10	461 489,228	458 400,041	458 400,501	458 400,615
	20	375 257,570	359 667,977	359 668,337	359 668,427
	30	325 806,302	282 646.635	282 646.919	282 646.99
	40	331029.613	249559.713	249559.963	249560.025
	50	389 279,374	277 650,050	277 650,329	277 650,398
	60	342 961,788	255 919,694	255 919,951	255 920,015
	70	327 205,078	277 951,853	277 952,131	277 952,201
	80	364 823,299	345 493,933	345 494,279	345 494,366
	90	442 956,859	438 627,226	438 627,665	438 627,775
	100	544 717,157	544 888,909	544 889,455	544 889,591

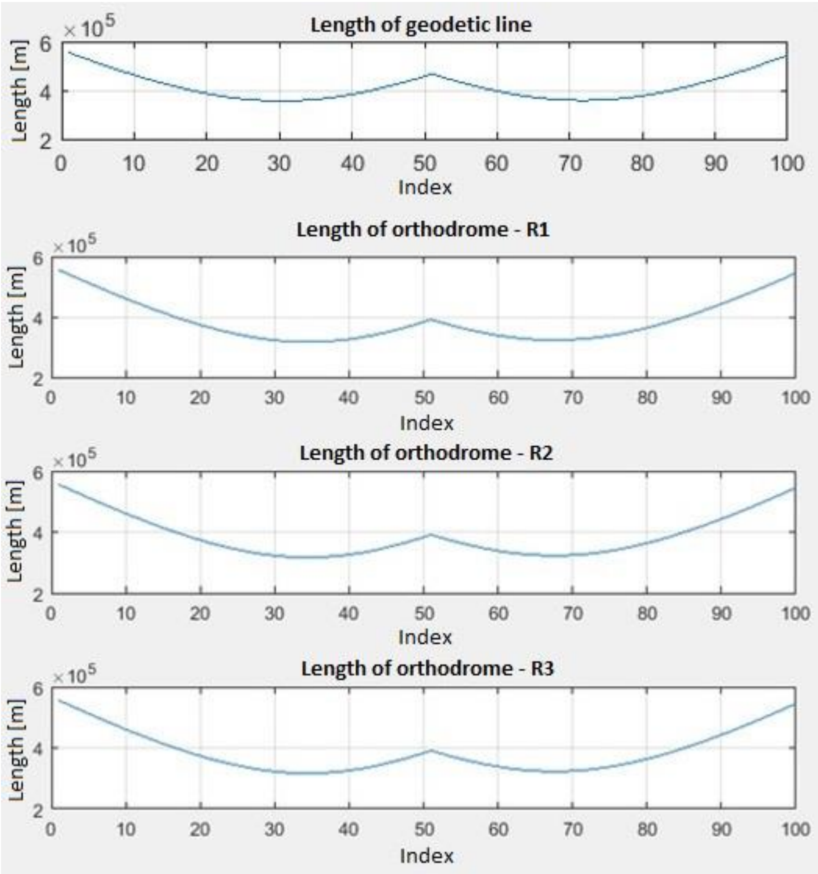

Figure 22. The lengths of geodetic lines and orthodromes for central point CT2

Table 4. Values of geodetic line lengths and orthodromes to individual peripheral points for CT2

	Point	Lengths of geodetic lines [m]	Lengths of orthodromes – R1 [m]	Lengths of orthodromes – R2 [m]	Lengths of orthodromes – R3 [m]
CT2 (Novi Sad)	1	554 873,071	555 974,702	555 975,259	555 975,398
	10	462 721,159	460 913,054	460 913,516	460 913,632
	20	386 593,210	373 781,406	373 781,781	373 781,875
	30	356 217,838	322 688,224	322 688,547	322 688,628
	40	383 059,727	326 083,765	326 084,092	326 084,174
	50	457 291,603	383 339,080	383 339,465	383 339,561
	60	396 447,927	338 564,512	338 564,851	338 564,936
	70	359 281,885	324 753,793	324 754,118	324 754,200
	80	377 645,642	363 817,212	363 817,577	363 817,668
	90	444 979,232	442 688,955	442 689,399	442 689,510
	100	543 426,978	544 916,152	544 916,699	544 916,835

Ajdabiya, at 28˚34'14''N and 19˚08'06''E, was selected as the third central point (CT3). At this latitude, the radius of curvature along the meridian of the GRS80 ellipsoid is M = 6 340 294,996 m, while along the first vertical N = 6 383 025,393 m.

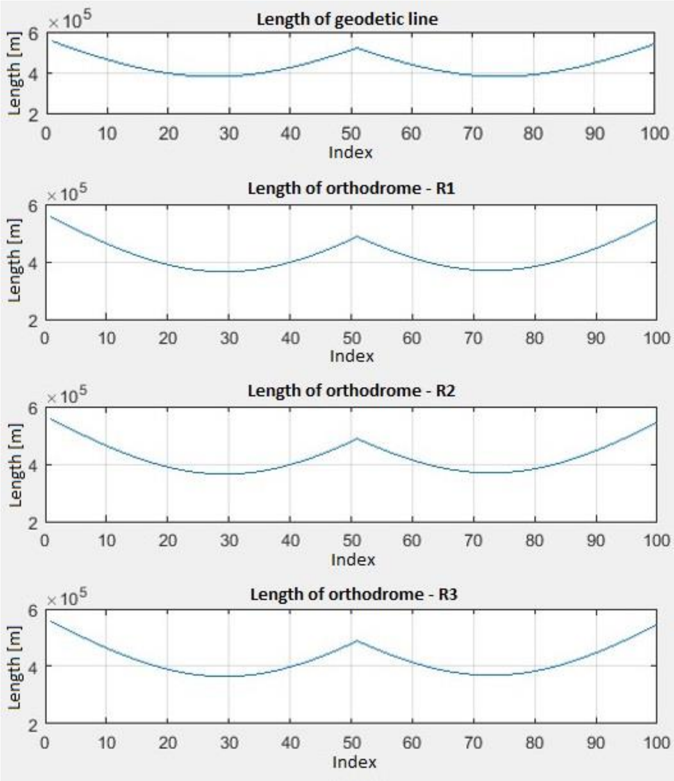

Figure 23. Lengths of geodetic lines and orthodromes for central point CT3

Table 5. Values of geodetic line lengths and orthodromes to individual peripheral points for CT3

	Point	Lengths of geodetic lines [m]	Lengths of orthodromes – R1 [m]	Lengths of orthodromes – R2 [m]	Lengths of orthodromes – R3 [m]
CT3 (Ajdabiya)	1	553 307,917	555 974,702	555 975,259	555 975,398
	10	463 472,879	463 955,172	463 955,637	463 955,753
	20	396 163,891	390 126,733	390 127,125	390 127,222
	30	381 140,550	364 811,178	364 811,544	364 811,635
	40	424 109,337	398 073,380	398 073,779	398 073,879
	50	510 558,643	478 375,668	478 376,148	478 376,267
	60	438 993,984	414 324,833	414 325,248	414 325,352
	70	385 903,029	371 344,354	371 344,726	371 344,819
	80	388 775,713	383 616,644	383 617,028	383 617,124
	90	446 607,274	447 224,898	447 225,347	447 225,459
	100	541 988,437	544 946,575	544 947,122	544 947,258

As expected, the geodetic line length values change with latitude (Figure 14). The diagram shows that the differences are minimized when the central and peripheral points are at approximately the same longitude (the meridian is a geodetic line), while the biggest differences are when the central and peripheral points are on the same parallel.

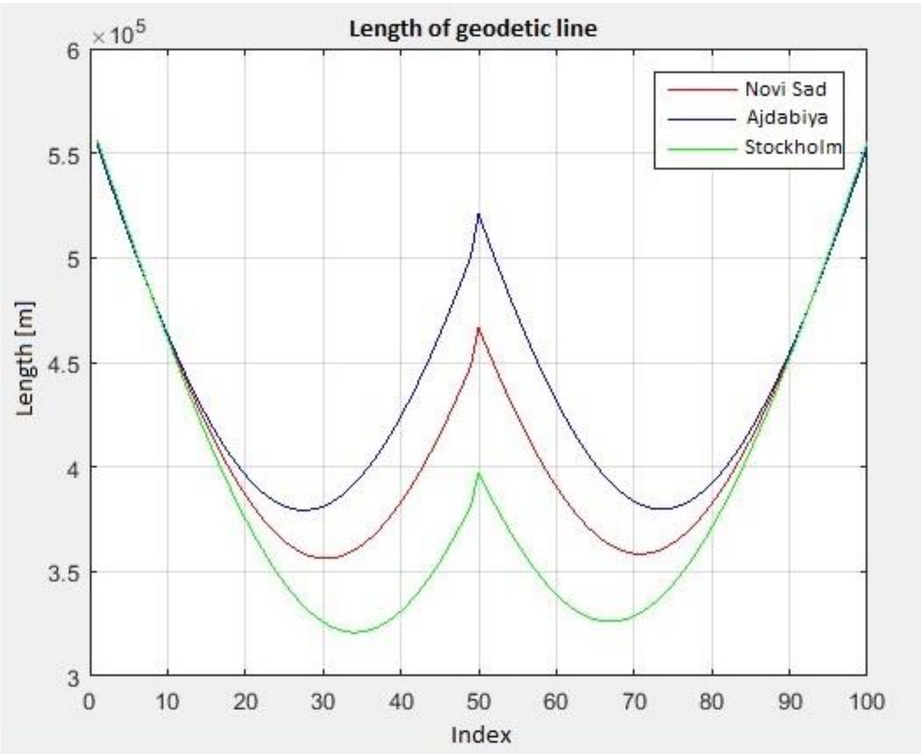

Figure 24. Changes in geodetic line lengths for three central points

4 DISCUSSION

Values of special interest were extracted from the obtained results. Table 4 shows the minimum lengths of geodetic lines, while table 5 shows the values of the minimum lengths of orthodromes.

Table 6. Minimum values of geodetic line lengths

Central point	Index	Minimal length of geodetic line [m]
CT1	34	320 814,648
CT2	30	356 217,838
CT3	28	379 344,067

Table 7. Minimum values of orthodrome length

Central point	Index	Minimum length of orthodrome for R1 [m]	Minimum length of orthodrome for R2 [m]	Minimum length of orthodrome for R3 [m]
CT1	40	249 559,713	249 559,963	249 560,025
CT2	34	316 920,828	316 921,146	316 921,225
CT3	29	364 698,810	364 699,176	364 699,267

Table 8. The difference in the length of the geodetic line and the orthodrome for CT1

	Index	Difference for orthodrome R1 [m]	Difference for orthodrome R2 [m]	Difference for orthodrome R3 [m]
CT1 (Stockholm)	1	177,874	177,317	177,178
	10	3 089,187	3 088,727	3 088,613
	20	15 589,592	15 589,232	15 589,142
	30	43 159,666	43 159,382	43 159,312
	40	81 469,900	81 469,650	81 469,587
	50	111 629,323	111 629,045	111 628,975
	60	87 042,094	87 041,837	87 041,773
	70	49 253,225	49 252,946	49 252,877
	80	19 329,366	19 329,019	19 328,933
	90	4 329,633	4 329,193	4 329,083
	100	-171,751	-172,298	-172,434

Table 9. The difference in the length of the geodetic line and the orthodrome for CT2

	Index	Difference for orthodrome R1 [m]	Difference for orthodrome R2 [m]	Difference for orthodrome R3 [m]
CT2 (Novi Sad)	1	-1 101,631	-1 102,188	-1 102,327
	10	1 808,104	1 807,642	1 807,527
	20	12 811,803	12 811,428	12 811,335
	30	33 529,614	33 529,291	33 529,209
	40	56 975,961	56 975,634	56 975,553
	50	73 952,523	73 952,138	73 952,042
	60	57 883,415	57 883,076	57 882,991
	70	34 528,092	34 527,766	34 527,685
	80	13 828,429	13 828,065	13 827,974
	90	2 290,277	2 289,833	2 289,722
	100	-1 489,174	-1 489,721	-1 489,857

In the tables, it can be seen that the minimum values of geodetic line lengths and orthodromes are not obtained for the same indices, and that the largest difference in indices is for the northernmost point, and the smallest for the southernmost central point.

Based on the calculated length values, approximation errors were also determined as the difference between the length of the geodetic line and the orthodrome for each central-peripheral point pair. The calculated values for every tenth index, as well as the maximum and mean error values are given in the following tables.

Table 10. The difference in the length of the geodetic line and the orthodrome for CT3

	Index	Difference for orthodrome R1 [m]	Difference for orthodrome R2 [m]	Difference for orthodrome R3 [m]
CT3 (Ajdabiya)	1	-2 666,784	-2 667,341	-2 667,481
	10	-482,292	-482,757	-482,873
	20	6 037,157	6 036,766	6 036,668
	30	16 329,371	16 329,006	16 328,914
	40	26 035,956	26 035,557	26 035,458
	50	32 182,975	32 182,495	32 182,376
	60	24 669,151	24 668,735	24 668,632
	70	14 558,675	14 558,302	14 558,209
	80	5 159,069	5 158,684	5 158,588
	90	-617,624	-618,072	-618,184
	100	-2 958,138	-2 958,684	-2 958,820

Table 11. The maximum values of the differences between the lengths of the geodetic line and the orthodrome

Central point	Index	Orthodrome R1 [m]	Orthodrome R2 [m]	Orthodrome R3 [m]
CT1	51	113 816,952	113 816,668	113 816,597
CT2	51	75 235,484	75 235,091	75 234,993
CT3	51	32 611,064	32 610,575	32 610,453

Table 12. Arithmetic means of the difference of lengths

Central point	Orthodrome R1 [m]	Orthodrome R2 [m]	Orthodrome R3 [m]
CT1	41 489,811	41 489,405	41 489,304
CT2	28 501,741	28 501,296	28 501,185
CT3	11 824,751	11 824,269	11 824,148

In Tables 6, 7 and 8, it can be observed that the deviation increases with decreasing latitude of the peripheral point and that it has the maximum value when the central and peripheral points are on the same parallel. After that, the deviation decreases, for one peripheral point it has a value of zero and then a negative value. At CT2 and CT3, the deviation for one peripheral point is 0 m and for peripheral points north of the central one, with the fact that at point CT3 that peripheral point is closer to the central one, viewed in terms of latitude. Bearing this in mind, as well as the fact that the maximum error is the smallest in CT3 and the largest in CT1 (almost 3,5 times larger), it follows that the spheres determined in the three presented ways are a significantly better approximation of the ellipsoid in areas closer to the equator than at higher latitudes. Moreover, if errors are viewed relatively, i.e., in relation to the corresponding values of the geodetic line lengths, for the peripheral point with index 50, the error value on CT3 will be 6 3% and on CT1 28 6%. Moreover, all three methods of calculating the radius of the sphere give similar deviation values, so this consideration is valid for all three spheres. The calculated deviation values for the sphere with radius R3 are also presented graphically in Figure 15. The diagram clearly shows that the smallest deviation is at the southernmost central point, as well as the smallest value variation.

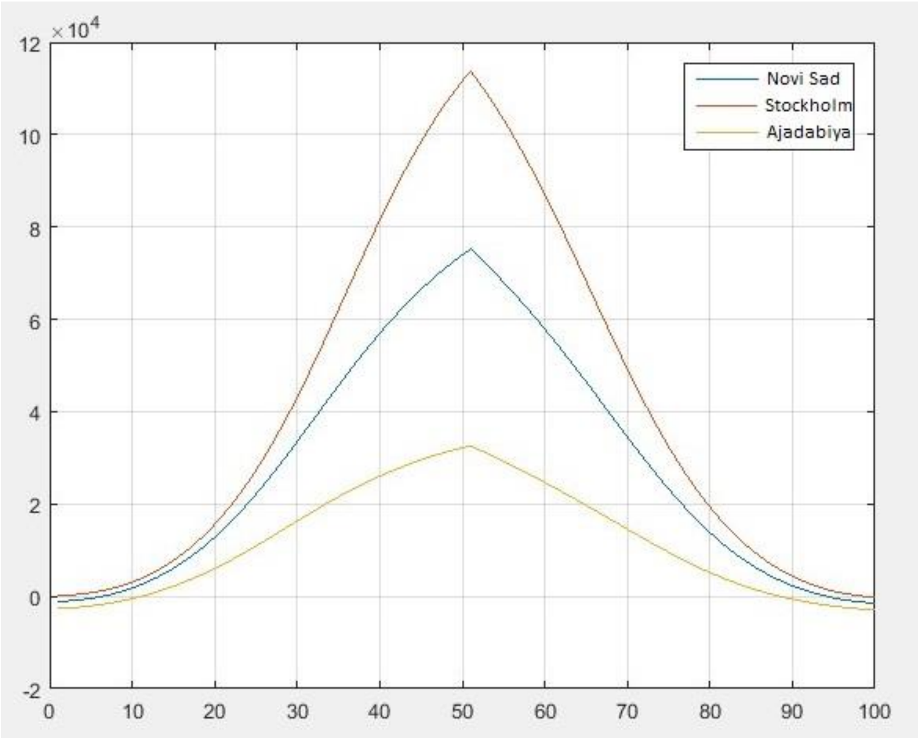

Figure 25. The values of the obtained differences for Stockholm, Novi Sad and Ajdabiya

From tables 6, 7 and 8, as well as from tables 9 and 10, it can be seen that the values of the differences vary very little when looking at the radii. For each central point and index, the differences are of the order of several decimeters. On a small-scale map (for which the sphere approximation is often used), these values are fractions of a millimeter. Compared to the difference values, these variations are several orders of magnitude smaller. All this leads to the conclusion that, from the point of view of accuracy, all three examined methods of approximation give almost identical results. However, from the values in the tables, it can be seen that the smallest deviation is always for the sphere of radius R3. Also, this radius is the simplest to calculate (arithmetic mean of three values), so although with modern computers it is not such a significant detail, it still follows that it is the best choice for approximation.

5 CONCLUSION

In this paper, three ways of approximating the ellipsoid with a sphere (equivalent volume, equivalent surface, and the arithmetic mean of the three semi-axes of the rotating ellipsoid) were examined. The difference between the length of the geodetic line and the length of the orthodrome on three spheres of different radii was taken as a measure of the accuracy of the approximation. Longitudes were calculated between the central and 100 peripheral points of different latitude and longitude. This calculation was repeated for three central points, at different latitudes.

The results showed that all three methods of calculating the radius of the sphere give very similar deviations, although the deviation for the radius calculated as an arithmetic mean was consistently the smallest. Since the sphere approximation is most often used for fine-scale maps, these variations in deviation are negligible. The calculated values also showed that these approximations give the best results for areas closer to the equator. Calculating with absolute errors, the sphere approximation at 30° latitude was 3,5 times better than at 60°, and if considering relative errors, it was 4,5 times better. In the era of modern geoinformation systems, computers with high processing power, and software tools for cartography, these values raise doubts about the necessity of using a sphere as the geometric shape that most closely approximates the Earth’s shape.

REFERENCES

[1] P. K. E. Vanicek, Geodezija - koncepti, New Brunswich: University of New Brunswich, 2005.

[2] B. Jovanović, Matematicka kartografija, Belgrade: Militarygeographic institute, 1983.

[3] A. Živković, Viša geodezija, Belgrade: University of Belgrade, 1972.

[4] M. V. M. K. A. P. V. Borisov, “Analiza svojstva geodetske linije u zavisnosti od položaja na elipsoidu,” *Izgradnja,* vol. 73, 2019.

[5] R. E. Deakin, Loxodrome on an ellipsoid, Melbourne: RMIT University, 2010.

[6] M. Fras, Študija ortodrome in loksodrome na krogli, elipsoidu ter v kartografskih projekcija, Ljubljana: Fakulteta za gradbeništvo in geodezijo, 2012.

[7] F. P. M. L. L. Č. K. S. D. Benković, Terestrička i elektronska navigacija, Split: Hidrografski institut ratne mornarice, 1986.

[8] M. B. B. D. S. Borisov, “Matematički modeli loksodrome i njihova primena,” *Tehnika - Naše građevinarstvo,* vol. 4, no. 65, 2011.

[9] N. D. I. Bjelovučić, “Priročnik za voditelja čolna,” Rogaška Slatina, 2007.

[10] D. H. Maling, Coordinate Systems and Map Projections, Pergamon Press, 1992.

[11] M. Lukovac, Karakteristične linije na elipsoidu i lopti, Novi Sad: University of Novi Sad - Faculty of technical sciences, 2013.